Towards compressive sensing imaging of real radio interferometric observations

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Outline

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Compressive sensing (CS)

- **"Nothing short of revolutionary."**
  – National Science Foundation
- Developed by Emmanuel Candes and David Donoho (and others).
- Next evolution of wavelet analysis.
- The mystery of JPEG compression (discrete cosine transform; wavelet transform).
- Acquisition versus imaging.

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An(other!) introduction to compressive sensing

- Linear operator (linear algebra) representation of signal decomposition:

$$x(t) = \sum_i \alpha_i \Psi_i(t) \quad \rightarrow \quad \boldsymbol{x} = \sum_i \Psi_i \alpha_i = \begin{pmatrix} | \\ \Psi_0 \\ | \end{pmatrix} \alpha_0 + \begin{pmatrix} | \\ \Psi_1 \\ | \end{pmatrix} \alpha_1 + \cdots \quad \rightarrow \quad \boxed{\boldsymbol{x} = \Psi \boldsymbol{\alpha}}$$

- Linear operator (linear algebra) representation of measurement:

$$y_i = \langle x, \Phi_j \rangle \quad \rightarrow \quad \boldsymbol{y} = \begin{pmatrix} - \ \Phi_0 \ - \\ - \ \Phi_1 \ - \\ \vdots \end{pmatrix} \boldsymbol{x} \quad \rightarrow \quad \boxed{\boldsymbol{y} = \Phi \boldsymbol{x}}$$

- Putting it together: $\boxed{\boldsymbol{y} = \Phi \boldsymbol{x} = \Phi \Psi \boldsymbol{\alpha}}$

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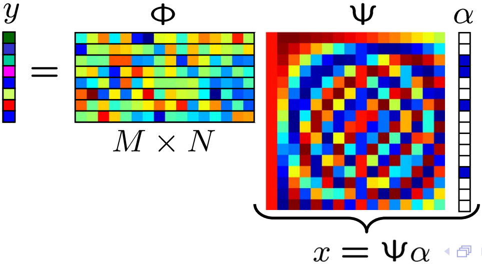

An(other!) introduction to compressive sensing

- Ill-posed inverse problem:

$$\boldsymbol{y} = \Phi \boldsymbol{x} + \boldsymbol{n} = \Phi \Psi \boldsymbol{\alpha} + \boldsymbol{n}.$$

- Solve by imposing a regularising prior that the signal to be recovered is sparse in Ψ, *i.e.* solve the following ℓ_0 optimisation problem:

$$\boldsymbol{\alpha}^\star = \arg\min_{\boldsymbol{\alpha}} \|\alpha\|_0 \text{ such that } \|\boldsymbol{y} - \Phi \Psi \boldsymbol{\alpha}\|_2 \leq \epsilon ,$$

  where the signal is synthesising by $\boldsymbol{x}^\star = \Psi \boldsymbol{\alpha}^\star$.

- Recall norms given by:

$$\|\alpha\|_0 = \text{no. non-zero elements} \qquad \|\alpha\|_1 = \sum_i |\alpha_i| \qquad \|\alpha\|_2 = \Big(\sum_i |\alpha_i|^2\Big)^{1/2}$$

- Solving this problem is difficult (combinatorial).
- Instead, solve the ℓ_1 optimisation problem (convex):

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- The solutions of the ℓ_0 and ℓ_1 problems are often the same.

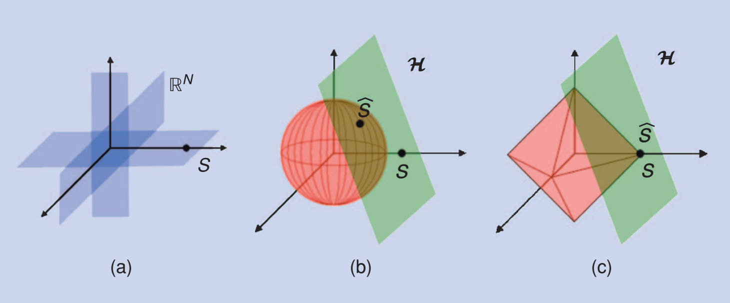

Figure: Geometry of (a) ℓ_0 (b) ℓ_2 and (c) ℓ_1 problems. [Credit: Baraniuk (2007)]

An(other!) introduction to compressive sensing

- In the absence of noise, compressed sensing is **exact!**
- Number of measurements required to achieve exact reconstruction is given by

$$M \geq c\mu^2 K \log N ,$$

where K is the sparsity and N the dimensionality.

- The coherence between the measurement and sparsity basis is given by

$$\mu = \sqrt{N} \max_{i,j} |\langle \Psi_i, \Phi_j \rangle| .$$

- Robust to noise.
- Many new developments (*e.g.* analysis vs synthesis, cosparsity, structured sparsity) and new applications.

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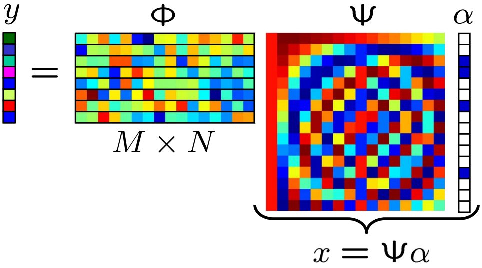

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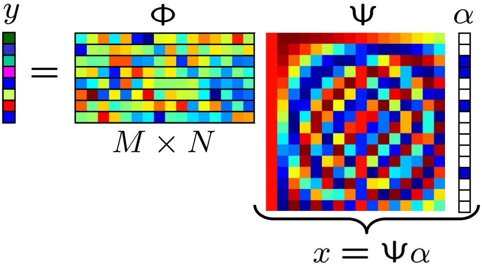

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Outline

Radio interferometric inverse problem

- Consider the ill-posed inverse problem of radio interferometric imaging:

$$\boxed{\boldsymbol{y} = \Phi \boldsymbol{x} + \boldsymbol{n}} \ ,$$

  where $\boldsymbol{y}$ are the measured visibilities, Φ is the linear measurement operator, $\boldsymbol{x}$ is the underlying image and $\boldsymbol{n}$ is instrumental noise.

- Measurement operator $\boxed{\Phi = \mathbf{M}\,\mathbf{F}\,\mathbf{C}\,\mathbf{A}}$ may incorporate:
  - primary beam $\mathbf{A}$ of the telescope;
  - w-component modulation $\mathbf{C}$ (responsible for the spread spectrum phenomenon);
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Interferometric imaging with compressed sensing

- Solve the interferometric imaging problem

$$\boldsymbol{y} = \Phi \boldsymbol{x} + \boldsymbol{n} \quad \text{with} \quad \Phi = \mathbf{M}\,\mathbf{F}\,\mathbf{C}\,\mathbf{A}\,,$$

  by applying a prior on sparsity of the signal in a sparsifying dictionary Ψ.

- Solve basis pursuit denoising problem

$$\boldsymbol{\alpha}^\star = \arg\min_{\boldsymbol{\alpha}} \|\alpha\|_1 \ \text{such that} \ \|\boldsymbol{y} - \Phi\Psi\boldsymbol{\alpha}\|_2 \leq \epsilon\,,$$

  where the image is synthesised by $\boldsymbol{x}^\star = \Psi\boldsymbol{\alpha}^\star$.

- Various choices for sparsifying dictionary Ψ, *e.g.* Dirac basis, Daubechies wavelets.
- Analysis versus synthesis problems, *e.g.* SARA algorithm (see following talk by Yves Wiaux).
- Recall the potential trade-off between sparsity and coherence.

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Review of the spread spectrum phenomenon

- The w-component modulation gives rise to the spread spectrum phenomenon first considered by Wiaux *et al.* (2009b).
- The w-component operator $\mathbf{C}$ has elements defined by

$$C(l,m) \equiv \exp\{i2\pi w\left(1-\sqrt{1-l^2-m^2}\right)\} \simeq \exp(i\pi w\|\boldsymbol{l}\|^2) \quad \text{for} \quad \|\boldsymbol{l}\|^4\, w \ll 1\,,$$

giving rise to to a linear chirp.

- For the (essentially) Fourier measurements of interferometric telescopes the coherence is the maximum modulus of the Fourier transform of the atoms of the sparsifying dictionary.
- Modulation by the chirp spreads the spectrum of the atoms of the sparsifying dictionary.
- Consequently, spreading the spectrum increases the incoherence between the sensing and sparsity bases, thus improving reconstruction fidelity.

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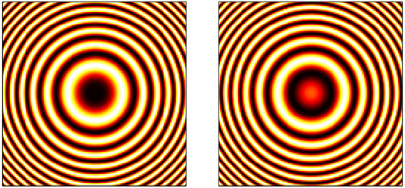

(a) Real part (b) Imaginary part

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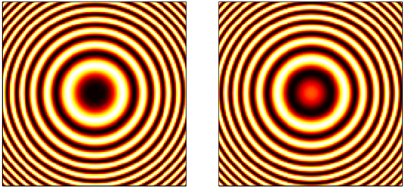

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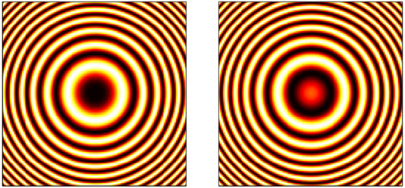

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Spread spectrum phenomenon for varying w

- Improved reconstruction fidelity of the spread spectrum phenomenon demonstrated with simulations by Wiaux *et al.* (2009b).
- However, previous analysis was restricted to fixed w for simplicity.
- Recently, we have examined the spread spectrum phenomenon for varying w.
- Work of Laura Wolz, in collaboration with JDM, Filipe Abdalla, Rafael Carrillo and Yves Wiaux.
- Apply the w-projection algorithm (Cornwell *et al.* 2008) to shift the chirp modulation through the Fourier transform:

$$\boxed{\Phi = \mathbf{M F C A} \quad \Rightarrow \quad \Phi = \mathbf{M \tilde{C} F A}} \, .$$

- Consider different w for each (u, v) and threshold each Fourier transformed chirp (each row of $\tilde{\mathbf{C}}$) to approximate $\tilde{\mathbf{C}}$ accurately by a sparse matrix.

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Spread spectrum phenomenon for varying w

- Perform simulations to assess the effectiveness of the spread spectrum phenomenon in the presence of varying w.
- Consider idealised simulations with uniformly random visibility sampling.

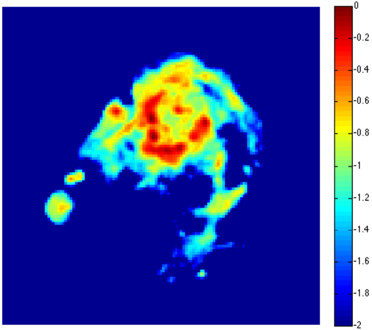
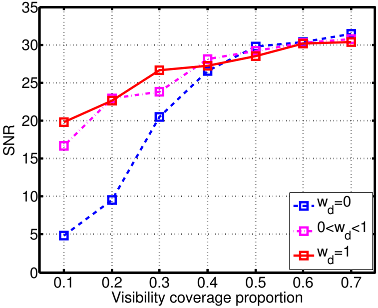


Figure: M31 (ground truth).

Spread spectrum phenomenon for varying w



(a) Daubechies 8 (Db8) wavelets

Figure: Reconstruction fidelity.

The improvement in reconstruction fidelity due to the spread spectrum phenomenon for varying w is almost as large as the case of constant maximum w!

- As expected, for the case where coherence is already optimal, there is little improvement.

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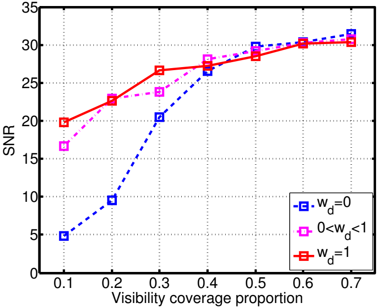

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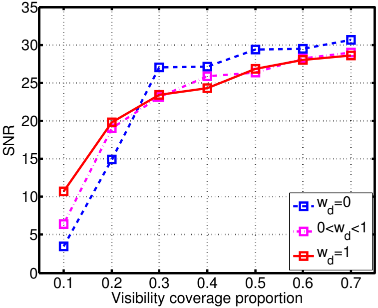

(b) Dirac basis

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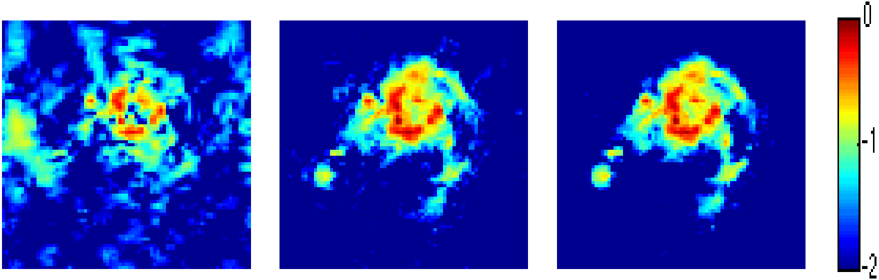

(a) $w_{\rm d} = 0 \rightarrow$ SNR$=$ 4.8dB (b) $w_{\rm d} \sim \mathcal{U}(0,1) \rightarrow$ SNR$=$ 16.7dB (c) $w_{\rm d} = 1 \rightarrow$ SNR$=$ 19.8dB

Figure: Reconstructed images for 10% coverage.

Outline

Supporting continuous visibilities

- Ideally we would like to model the continuous Fourier transform operator

$$\Phi = \mathbf{F}^{c} \, .$$

- But this is slow!
- We have incorporated gridding into our CS interferometric imaging framework.
- Work of Rafael Carrillo, in collaboration with Yves Wiaux and JDM.
- Model with the measurement operator

$$\boxed{\Phi = \mathbf{G} \, \mathbf{F} \, \mathbf{Z} \, \mathbf{D}} \, ,$$

  where we incorporate:

  - convolutional gridding operator $\mathbf{G}$;
  - fast Fourier transform $\mathbf{F}$;
  - zero-padding operator $\mathbf{Z}$ to upsample the discrete visibility space;
  - normalisation operator $\mathbf{D}$ to undo the convolution gridding (reciprocal of the inverse Fourier transform of the gridding kernel).

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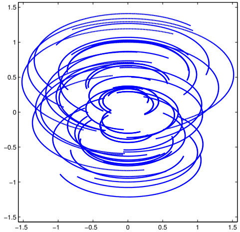
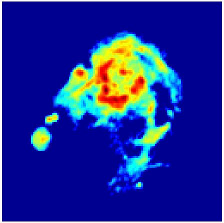
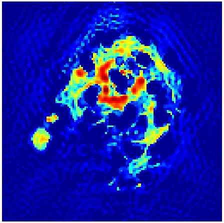
Reconstruction with continuous visibilities



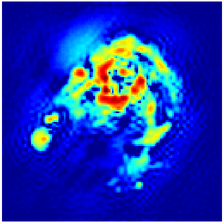
(a) Coverage



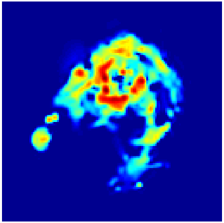
(b) M31 (ground truth)



(c) Dirac basis → SNR= 8.2dB



(d) Db8 wavelets → SNR= 11.1dB



(e) SARA → SNR= 13.4dB

Figure: Reconstructed images from continuous visibilities.

Summary

- Effectiveness of compressive sensing for radio interferometric imaging demonstrated already (Wiaux *et al.* 2009a, Wiaux *et al.*2009b, Wiaux *et al.* 2009c, McEwen & Wiaux 2011, Carrillo *et al.* 2012).
- Provide improvements in reconstruction fidelity, flexibility and computation time.
- Important to take these methods to the realistic setting so that their advantages can be realised on observations made by real radio interferometric telescopes.

- Taken first steps toward more realistic setting.
- Studied the spread spectrum phenomenon for varying w.
- The improvement in reconstruction fidelity due to the spread spectrum phenomenon for varying w is almost as large as the case of constant maximum w!
- Incorporated a gridding operator into our framework to support continuous visibilities.

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Outlook

- BUT... so far we remain idealised.
- We (Rafael Carrillo, JDM and Yves Wiaux) are developing an optimised C code (`PURIFY`) to scale to the realistic setting.
- Preliminary tests indicate that this code provides in excess of an order of magnitude speed improvement and supports scaling to very large data-sets.