A novel sampling theorem on the sphere with implications for compressive sampling

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Outline

1. Harmonic analysis on the sphere
2. A novel sampling theorem
3. Compressive sensing
4. Summary
Consider the space of square integrable functions on the sphere $L^2(S^2)$, with the inner product of $f, g \in L^2(S^2)$ defined by

$$\langle f, g \rangle = \int_{S^2} d\Omega(\theta, \varphi) f(\theta, \varphi) g^*(\theta, \varphi),$$

where $d\Omega(\theta, \varphi) = \sin \theta \, d\theta \, d\varphi$ is the usual invariant measure on the sphere and $(\theta, \varphi)$ define spherical coordinates with colatitude $\theta \in [0, \pi]$ and longitude $\varphi \in [0, 2\pi)$. Complex conjugation is denoted by the superscript $^*$. 

The scalar spherical harmonic functions form the canonical orthogonal basis for the space of $L^2(S^2)$ scalar functions on the sphere and are defined by

$$Y_{\ell m}(\theta, \varphi) = \sqrt{\frac{2\ell + 1}{4\pi} \frac{(\ell - m)!}{(\ell + m)!}} P^m_\ell(\cos \theta) \, e^{im\varphi},$$

for natural $\ell \in \mathbb{N}$ and integer $m \in \mathbb{Z}$, $|m| \leq \ell$, where $P^m_\ell(x)$ are the associated Legendre functions.

Eigenfunctions of the Laplacian on the sphere: $\Delta_{S^2} Y_{\ell m} = -\ell(\ell + 1) Y_{\ell m}$.

Orthogonality relation: $\langle Y_{\ell m}, Y_{\ell' m'} \rangle = \delta_{\ell \ell'} \delta_{mm'}$, where $\delta_{ij}$ is the Kronecker delta symbol.

Completeness relation:

$$\sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} Y_{\ell m}(\theta, \varphi) Y^*_{\ell m}(\theta', \varphi') = \delta(\cos \theta - \cos \theta') \delta(\varphi - \varphi'),$$

where $\delta(x)$ is the Dirac delta function.
Spherical harmonics

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where \( \delta(x) \) is the Dirac delta function.
Spherical harmonic transform

- Any square integrable scalar function on the sphere \( f \in L^2(S^2) \) may be represented by its spherical harmonic expansion:

\[
  f(\theta, \varphi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} f_{\ell m} Y_{\ell m}(\theta, \varphi).
\]

- The spherical harmonic coefficients are given by the usual projection onto each basis function:

\[
  f_{\ell m} = \langle f, Y_{\ell m} \rangle = \int_{S^2} d\Omega(\theta, \varphi) f(\theta, \varphi) Y_{\ell m}^*(\theta, \varphi).
\]

- We consider signals on the sphere band-limited at \( L \), that is signals such that \( f_{\ell m} = 0, \forall \ell \geq L \) \( \Rightarrow \) summations may be truncated to \( L - 1 \).

- Aside: Generalise to spin functions on the sphere.
  Square integrable spin functions on the sphere \( s f \in L^2(S^2) \), with integer spin \( s \in \mathbb{Z} \), \( |s| \leq \ell \), are defined by their behaviour under local rotations. By definition, a spin function transforms as

\[
  s f'(\theta, \varphi) = e^{-is\chi} s f(\theta, \varphi)
\]

under a local rotation by \( \chi \), where the prime denotes the rotated function.

- Sampling theorems on the sphere.
Spherical harmonic transform

- Any square integrable scalar function on the sphere \( f \in L^2(S^2) \) may be represented by its **spherical harmonic expansion**:

\[
f(\theta, \varphi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} f_{\ell m}(\theta, \varphi) Y_{\ell m}(\theta, \varphi) .
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sf'(\theta, \varphi) = e^{-is\chi} sf(\theta, \varphi)
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Square integrable spin functions on the sphere \( sf \in L^2(S^2) \), with integer spin \( s \in \mathbb{Z}, |s| \leq \ell \), are defined by their behaviour under local rotations. By definition, a spin function transforms as

\[
sf'(\theta, \varphi) = e^{-is\chi} sf(\theta, \varphi)
\]

under a local rotation by \( \chi \), where the prime denotes the rotated function.

- Sampling theorems on the sphere.
Inexact spherical harmonic transforms exist for a variety of pixelisations of the sphere, for example:

- **HEALpix** *(Gorski et al. 2005)*
- **IGLOO** *(Crittenden & Turok 1998)*

→ **Do not lead to sampling theorems on the sphere!**

- **Driscoll & Healy** (1994) sampling theorem:
  - Equiangular pixelisation of the sphere
  - Require $\sim 4L^2$ samples on the sphere
  - Semi-naive algorithm with complexity $O(L^3)$
    (algorithms with lower scaling exist but they are not generally stable)
  - Require a precomputation or otherwise restricted use of Wigner recursions

- **Gauss-Legendre** sampling theorem:
  - Sample positions given by roots of Legendre functions
  - Require $\sim 2L^2$ samples on the sphere
  - Simple separation of variables gives algorithm with complexity $O(L^3)$
  - Require a precomputation or otherwise restricted use of Wigner recursions
Sampling theorems on the sphere: state-of-the-art

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A novel sampling theorem on the sphere

- We have developed a new sampling theorem and corresponding fast algorithms by performing a factoring of rotations and then by associating the sphere with the torus through a periodic extension.

- Similar (in flavour but not detail!) to making a periodic extension in $\theta$ of a function $f$ on the sphere.
A novel sampling theorem on the sphere

- We have developed a new sampling theorem and corresponding fast algorithms by performing a factoring of rotations and then by associating the sphere with the torus through a periodic extension.

- Similar (in flavour but not detail!) to making a periodic extension in $\theta$ of a function $f$ on the sphere.

Figure: Associating functions on the sphere and torus
By a factoring of rotations, a reordering of summations and a separation of variables, the inverse transform of $s f$ may be written:

Inverse spherical harmonic transform

\[ s f(\theta, \varphi) = \sum_{m=-L-1}^{L-1} s F_m(\theta) e^{im\varphi} \]

\[ s F_m(\theta) = \sum_{m'=-L-1}^{L-1} s F_{m'm'} e^{im'\theta} \]

\[ s F_{m'm'} = (-1)^s i^{-(m+s)} \sum_{\ell=0}^{L-1} \sqrt{\frac{2\ell + 1}{4\pi}} \Delta^\ell_{m'm'} \Delta^\ell_{m'-s} s f_{\ell m} \]

where $\Delta^\ell_{mn} \equiv d^\ell_{mn}(\pi/2)$ are the reduced Wigner functions evaluated at $\pi/2$. 
A novel sampling theorem on the sphere: forward transform

By a factoring of rotations, a reordering of summations and a separation of variables, the forward transform of $s_f$ may be written:

**Forward spherical harmonic transform**

$$s_f \ell_m = (-1)^s i^{m+s} \sqrt{\frac{2\ell + 1}{4\pi}} \sum_{m'=-L}^{L-1} \Delta_{m' m} \Delta_{m' \ell}, -s G_{mm'}$$

$$s G_{mm'} = \int_0^\pi d\theta \sin \theta s G_m(\theta) e^{-im' \theta}$$

$$s G_m(\theta) = \int_0^{2\pi} d\varphi s f(\theta, \varphi) e^{-im \varphi}$$

- This formulation highlights similarities with Fourier series representation.
- The Fourier series expansion is only defined for periodic functions; thus, to recast these expressions in a form amenable to the application of Fourier transforms we must make a periodic extension in colatitude $\theta$. 
A novel sampling theorem on the sphere: forward transform

By a factoring of rotations, a reordering of summations and a separation of variables, the forward transform of \( s f \) may be written:

\[
\mathcal{F}_{\ell m} = (-1)^s i^{m+s} \sqrt{\frac{2\ell + 1}{4\pi}} \sum_{m' = -(L-1)}^{L-1} \Delta_{m'm}^{\ell} \Delta_{m'}^{\ell}, -s \, s G_{mm'}
\]

\[
s G_{mm'} = \int_0^\pi d\theta \sin \theta \, s G_m(\theta) \, e^{-im'\theta}
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s G_m(\theta) = \int_0^{2\pi} d\varphi \, s f(\theta, \varphi) \, e^{-im\varphi}
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This formulation highlights similarities with Fourier series representation.

The Fourier series expansion is only defined for periodic functions; thus, to recast these expressions in a form amenable to the application of Fourier transforms we must make a periodic extension in colatitude \( \theta \).
A novel sampling theorem on the sphere: properties

Properties of our new sampling theorem:
- Equiangular pixelisation of the sphere
- Require $\sim 2L^2$ samples on the sphere (and still fewer than Gauss-Legendre sampling)
- Exploit fast Fourier transforms to yield a fast algorithm with complexity $\mathcal{O}(L^3)$
- No precomputation and very flexible regarding use of Wigner recursions
- Extends to spin function on the sphere with no change in complexity or computation time

Figure: Performance of our sampling theorem (MW=red; DH=green; GL=blue)
A novel sampling theorem on the sphere: quadrature

- Sampling theorems effectively encode (often implicitly) an exact quadrature rule for evaluating the integral of a band-limited function on the sphere.

- The quadrature rule can be made explicit:

\[
\int_{S^2} d\Omega(\theta, \varphi) \, sf(\theta, \varphi) = \sum_{t=0}^{L-1} \sum_{p=0}^{2L-2} q_{MW}(\theta_t) \, sf(\theta_t, \varphi_p).
\]

- A similar quadrature rule can be given for the Driscoll & Healy sampling theorem. However, \(2L\) samples in colatitude \(\theta\) are required \(\Rightarrow \sim 4L^2\) samples on the sphere.
A reduction in the number of samples required to represent a band-limited signal on the sphere has important implications for compressive sensing.

Many natural signals are sparse in measures defined in the spatial domain, such as in the magnitude of their gradient.

A more efficient sampling of a band-limited signal on the sphere improves both the dimensionality and sparsity of the signal in the spatial domain.

For a given number of measurements, a more efficient sampling theorem improves the quality of compressive sampling reconstruction.

Illustrate with a total variation (TV) inpainting problem on the sphere.
Consider inpainting problem $y = \Phi x + n$ in the context of different sampling theorems, where:

- the samples of $f$ are denoted by the concatenated vector $x \in \mathbb{R}^N$;
- $N$ is the number of samples on the sphere of the chosen sampling theorem;
- $M$ noisy measurements $y \in \mathbb{R}^M$ are acquired;
- the measurement operator $\Phi \in \mathbb{R}^{M \times N}$ represents a random masking of the signal;
- the noise $n \in \mathbb{R}^M$ is assumed to be iid Gaussian with zero mean.

Define TV norm on the sphere:

$$
\int_{S^2} d\Omega \, |\nabla f| \simeq \sum_{t=0}^{N_\theta-1} \sum_{p=0}^{N_\phi-1} |\nabla f| q(\theta_t) \simeq \sum_{t=0}^{N_\theta-1} \sum_{p=0}^{N_\phi-1} \sqrt{q^2(\theta_t)(\delta_\theta x)^2 + \frac{q^2(\theta_t)}{\sin^2 \theta_t} (\delta_\phi x)^2} \equiv \|x\|_{TV}.
$$

TV inpainting problem solved directly on the sphere:

$$
x^* = \arg \min_x \|x\|_{TV} \text{ such that } \|y - \Phi x\|_2 \leq \epsilon.
$$

TV inpainting problem solved in harmonic space:

$$
\hat{x}^* = \arg \min_{\hat{x}} \|\Lambda \hat{x}\|_{TV} \text{ such that } \|y - \Phi \Lambda \hat{x}\|_2 \leq \epsilon,
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where $\Lambda$ represents the inverse spherical harmonic transform and harmonic coefficients are represented by the concatenated vector $\hat{x} \in \mathbb{C}^{L^2}$.
Consider inpainting problem $y = \Phi x + n$ in the context of different sampling theorems, where:

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Define TV norm on the sphere:

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\int_{S^2} d\Omega \, |\nabla f| \simeq \sum_{t=0}^{N_{\theta} - 1} \sum_{p=0}^{N_{\varphi} - 1} |\nabla f| \, q(\theta_t) \simeq \sum_{t=0}^{N_{\theta} - 1} \sum_{p=0}^{N_{\varphi} - 1} \sqrt{q^2(\theta_t)(\delta_{\theta}x)^2 + \frac{q^2(\theta_t)}{\sin^2 \theta_t}(\delta_{\varphi}x)^2} \equiv \|x\|_{TV}.
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**TV inpainting**

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- Define TV norm on the sphere:

  $\int_{S^2} d\Omega |\nabla f| \simeq \sum_{t=0}^{N\theta - 1} \sum_{p=0}^{N\varphi - 1} |\nabla f| q(\theta_t) \simeq \sum_{t=0}^{N\theta - 1} \sum_{p=0}^{N\varphi - 1} \sqrt{q^2(\theta_t)(\delta \theta x)^2 + \frac{q^2(\theta_t)}{\sin^2 \theta_t}(\delta \varphi x)^2} \equiv \|x\|_{TV}$

- TV inpainting problem solved directly on the sphere:

  $x^* = \arg \min_x \|x\|_{TV}$ such that $\|y - \Phi x\|_2 \leq \epsilon$.

- TV inpainting problem solved in harmonic space:

  $\hat{x}^* = \arg \min_{\hat{x}} \|\Lambda \hat{x}\|_{TV}$ such that $\|y - \Phi \Lambda \hat{x}\|_2 \leq \epsilon$,

where $\Lambda$ represents the inverse spherical harmonic transform and harmonic coefficients are represented by the concatenated vector $\hat{x} \in \mathbb{C}^{L^2}$.
TV inpainting

- Solve TV inpainting problem on the sphere in the context of the Driscoll & Healy sampling theorem and our new sampling theorem.

Figure: Earth topographic data reconstructed in the harmonic domain for $M/L^2 = 1/2$
Solve TV inpainting problem on the sphere in the context of the Driscoll & Healy sampling theorem and our new sampling theorem.

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Figure: Earth topographic data reconstructed in the harmonic domain for $M/L^2 = 1/2$
Figure: Reconstruction performance for the DH and MW sampling theorems
We have developed a new sampling theorem on the sphere requiring fewer than half the number of samples of the canonical Driscoll & Healy sampling theorem.

A reduction in the number of samples required to represent a band-limited signal on the sphere has important implications for compressive sensing, both in terms of the dimensionality and sparsity of signals.

We have demonstrated improved reconstruction quality when solving an inpainting problem in the context of different sampling theorems.

Upcoming publications


SSHT code

Code to compute exact spin spherical harmonic transforms (SSHT) in the context of our new sampling theorem will be available very soon from:

http://www.jasonmcewen.org/