Scalable and Equivariant Spherical CNNs by Discrete-Continuous (DISCO) Convolutions
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Jeremy Ocampo, Matthew A. Price, & Jason D. McEwen*

* Corresponding author: jason.mcewen@kagenova.com, www.jasonmcewen.org

Kagenova Limited, UCL
www.kagenova.com, www.ucl.ac.uk

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Whenever observe over angles, recover data on 2D sphere (or 3D rotation group).

Construct **CNNs natively on the sphere** and encode **rotational equivariance**.
Categorization of spherical CNNs frameworks

Continuous

✔️ Equivariant
❌ Not Scalable

Discrete

❌ Not Equivariant
✔️ Scalable


Categorization of spherical CNNs frameworks

Continuous

- Equivariant
- Not Scalable


Discrete

- Not Equivariant
- Scalable


Discrete-Continuous (DISCO) [this work]

- Equivariant
- Scalable

(Ocampo, Price & McEwen 2023)
Scalable and Equivariant Spherical CNNs by Discrete-Continuous (DISCO) Convolutions
(Ocampo, Price & McEwen 2023; arXiv:2209.13603)

Follows by a careful hybrid representation of the spherical convolution:

- some components left continuous, to facilitate accurate rotational equivariance;
- while other components are discretized, to yield scalable computation.
Discrete-continuous (DISCO) spherical convolution

Spherical convolution can be carefully approximated by the DISCO representation

$$(f \star \psi)(R) = \int_{S^2} f(\omega) \psi(R^{-1}\omega) d\omega \approx \sum_i f[\omega_i] \psi(R^{-1}\omega_i) q(\omega_i),$$

for spherical signal and filter kernel $f, \psi : S^2 \rightarrow \mathbb{R}$, with spherical coordinates $\omega \in S^2$, where, for now, we consider 3D rotations $R \in \text{SO}(3)$.

- Appeal to sampling theorem on the sphere with quadrature weights $q : S^2 \rightarrow \mathbb{R}$ (McEwen & Wiaux 2011; arXiv:1110.6298):
  - all information content of signal captured by samples $\{f[\omega_i]\}_i$;
  - continuous integral evaluated accurately by quadrature (exact for sufficient sampling).
- Filter $\psi$ and rotation $R$ treated continuously to avoid any discretization artefacts.
Restricting rotations to SO(3)/SO(2)

While the DISCO spherical convolution is already efficient, we seek further computational savings by restricting the space of rotations to quotient space SO(3)/SO(2).

- Analogous to Euclidean planar CNNs, where filters are translated across the image but are not rotated in the plane.

- However, as the space SO(3)/SO(2) is not a group, when restricting rotations in this manner important differences to the usual setting arise.

\[ R = Z(\alpha)Y(\beta)Z(\gamma) \in SO(3) \]

\[ R = Z(\alpha)Y(\beta) \in SO(3)/SO(2) \simeq S^2 \]
DISCO spherical convolution \( f \star \psi \) for rotations \( Q, R \in \text{SO}(3) \) satisfies \( \text{SO}(3) \) rotational equivariance:

\[
((Qf) \star \psi)(R) = (Q(f \star \psi))(R).
\]

Only holds since \( \text{SO}(3) \) exhibits a group structure and so \( Q^{-1}R \in \text{SO}(3) \).
Asymptotic rotational equivariance for rotations $R \in \text{SO}(3)/\text{SO}(2)$

DISCO spherical convolution $f \circledast \psi$ for rotations $Q, R \in \text{SO}(3)/\text{SO}(2)$ does not satisfy $\text{SO}(3)$ or $\text{SO}(3)/\text{SO}(2)$ rotational equivariance (in contrast to the Euclidean setting).

But DISCO spherical convolution $f \circledast \psi$ does satisfy asymptotic $\text{SO}(3)$ equivariance as $\beta \to 0$, where $Q = Z(\alpha)Y(\beta)Z(\gamma)$.

Asymptotic $\text{SO}(3)$ equivariance of significant practical use since content in spherical signals often orientated and similar content appears at similar latitudes, particularly for $360^\circ$ panoramic photos and video.
DISCO convolution affords a computationally scalable implementation.

1. Spare tensor representation.
2. Memory compression.
3. Custom sparse gradients.

Linear scaling in number of pixels on the sphere $O(N) = O(L^2)$ for both computational cost and memory usage.
DISCO spherical CNNs exhibit excellent rotational equivariance and are highly computationally scalable, supporting high-resolution input/output data for dense-prediction tasks.
Example predictions for depth estimation of Pano3D data (depth plotted in meters).
# Depth estimation for Pano3D

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameters</th>
<th>Depth Error Metrics</th>
<th>Depth Accuracy Metrics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planar UNet</td>
<td>27M</td>
<td>wRMSE: 0.4520, wRMSLE: 0.1300, wAbsRel: 0.1147, wSqRel: 0.0811</td>
<td>$\delta_{1.05}$: 36.68%, $\delta_{1.1}$: 60.59%, $\delta_{1.25}$: 88.31%, $\delta_{1.25^2}$: 96.96%</td>
</tr>
<tr>
<td>DISCO-Directional</td>
<td>658k</td>
<td>wRMSE: 0.5063, wRMSLE: 0.1695, wAbsRel: 0.1109, wSqRel: 0.0852</td>
<td>$\delta_{1.05}$: 38.32%, $\delta_{1.1}$: 62.12%, $\delta_{1.25}$: 88.65%, $\delta_{1.25^2}$: 97.29%</td>
</tr>
<tr>
<td>(Ours)</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
Semantic segmentation for 2D3Ds dataset

Example predictions for semantic segmentation of 2D3DS data.
### Semantic segmentation for 2D3Ds dataset

<table>
<thead>
<tr>
<th>Model</th>
<th>mIoU</th>
<th>mAcc</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planar UNet</td>
<td>35.9</td>
<td>50.8</td>
</tr>
<tr>
<td>UGSCNN</td>
<td>38.3</td>
<td>54.7</td>
</tr>
<tr>
<td>GaugeNet</td>
<td>39.4</td>
<td>55.9</td>
</tr>
<tr>
<td>HexRUNet</td>
<td>43.3</td>
<td>58.6</td>
</tr>
<tr>
<td>SWSCNNs</td>
<td>43.4</td>
<td>58.7</td>
</tr>
<tr>
<td>CubeNet</td>
<td>45.0</td>
<td>62.5</td>
</tr>
<tr>
<td>MöbiusConv</td>
<td>43.3</td>
<td>60.9</td>
</tr>
<tr>
<td>TangentImg</td>
<td>41.8</td>
<td>54.9</td>
</tr>
<tr>
<td>HoHoNet</td>
<td>43.3</td>
<td>53.9</td>
</tr>
<tr>
<td>DISCO-Directional-Aug (Ours)</td>
<td>45.7</td>
<td>62.7</td>
</tr>
</tbody>
</table>
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Can scale rotationally equivariant spherical CNNs to high-resolution input/output data for dense-prediction tasks.

SOTA performance on all benchmark problems considered to date.

Code available on request at https://kagenova.com/products/copernicAI/ or contact jason.mcewen@kagenova.com.