Intrinsic advantages of the w component and spherical imaging for wide-field radio interferometry

Jason McEwen[1] & Yves Wiaux[2]
[1] http://www.jasonmcewen.org
[2] http://people.epfl.ch/wiaux

Presented by Anna Scaife

BASP research node
Institute of Electrical Engineering,
Ecole Polytechnique Fédérale de Lausanne (EPFL), Switzerland

URSI GASS 2011 :: Istanbul

Outline

Radio interferometric imaging

- **Interferometric imaging:**
  recover an image from noisy and incomplete Fourier measurements.
- Resulting **ill-posed inverse problem** is described by

$$\boldsymbol{y} = \Phi \boldsymbol{x} + \boldsymbol{n} \, ,$$

with:

  - incomplete Fourier measurements taken by the interferometer $\boldsymbol{y}$;
  - linear measurement operator Φ;
  - underlying image $\boldsymbol{x}$;
  - noise $\boldsymbol{n}$.

- **Measurement operator** Φ incorporates:
  - primary beam of the telescope;
  - w component modulation responsible for the spread spectrum phenomenon;
  - Fourier transform;
  - masking which encodes the incomplete measurements taken by the interferometer.

Radio interferometric imaging

- **Interferometric imaging:**
  recover an image from noisy and incomplete Fourier measurements.
- Resulting **ill-posed inverse problem** is described by

$$y = \Phi x + n \ ,$$

with:
  - incomplete Fourier measurements taken by the interferometer y;
  - linear measurement operator Φ;
  - underlying image x;
  - noise n.
- **Measurement operator** Φ incorporates:
  - primary beam of the telescope;
  - w component modulation responsible for the spread spectrum phenomenon;
  - Fourier transform;
  - masking which encodes the incomplete measurements taken by the interferometer.

Radio interferometric imaging with compressed sensing

- Solved by applying a **prior on sparsity** of the signal in a sparsifying basis Ψ or in the magnitude of its gradient.
- Image is recovered by solving:
  - Basis Pursuit denoising problem

    $$\alpha^\star = \arg\min_{\alpha} \|\alpha\|_1 \text{ such that } \|y - \Phi\Psi\alpha\|_2 \leq \epsilon ,$$

    where the image is synthesising by $x^\star = \Psi\alpha^\star$;
  - Total Variation (TV) denoising problem

    $$x^\star = \arg\min_{x} \|x\|_{TV} \text{ such that } \|y - \Phi x\|_2 \leq \epsilon .$$

- ℓ_1-norm $\|\cdot\|_1$ is given by the sum of the absolute values of the signal.
- TV norm $\|\cdot\|_{TV}$ is given by the ℓ_1-norm of the gradient of the signal.
- Tolerance ϵ is related to an estimate of the noise variance.

Radio interferometric imaging with compressed sensing

- Solved by applying a **prior on sparsity** of the signal in a sparsifying basis Ψ or in the magnitude of its gradient.
- Image is recovered by solving:
  - Basis Pursuit denoising problem

$$\boldsymbol{\alpha}^\star = \arg\min_{\boldsymbol{\alpha}} \|\alpha\|_1 \text{ such that } \|\boldsymbol{y} - \Phi\Psi\boldsymbol{\alpha}\|_2 \leq \epsilon ,$$

    where the image is synthesising by $\boldsymbol{x}^\star = \Psi\boldsymbol{\alpha}^\star$;
  - Total Variation (TV) denoising problem

$$\boldsymbol{x}^\star = \arg\min_{\boldsymbol{x}} \|\boldsymbol{x}\|_{\text{TV}} \text{ such that } \|\boldsymbol{y} - \Phi\boldsymbol{x}\|_2 \leq \epsilon .$$

- ℓ_1-norm $\|\cdot\|_1$ is given by the sum of the absolute values of the signal.
- TV norm $\|\cdot\|_{\text{TV}}$ is given by the ℓ_1-norm of the gradient of the signal.
- Tolerance ϵ is related to an estimate of the noise variance.

Radio interferometric imaging with compressed sensing

- Solved by applying a **prior on sparsity** of the signal in a sparsifying basis Ψ or in the magnitude of its gradient.
- Image is recovered by solving:
  - Basis Pursuit denoising problem

$$\boldsymbol{\alpha}^\star = \arg\min_{\boldsymbol{\alpha}} \|\alpha\|_1 \;\text{ such that }\; \|\boldsymbol{y} - \Phi\Psi\boldsymbol{\alpha}\|_2 \leq \epsilon \;,$$

    where the image is synthesising by $\boldsymbol{x}^\star = \Psi\boldsymbol{\alpha}^\star$;
  - Total Variation (TV) denoising problem

$$\boldsymbol{x}^\star = \arg\min_{\boldsymbol{x}} \|\boldsymbol{x}\|_{\mathrm{TV}} \;\text{ such that }\; \|\boldsymbol{y} - \Phi\boldsymbol{x}\|_2 \leq \epsilon \;.$$

- ℓ_1-norm $\|\cdot\|_1$ is given by the sum of the absolute values of the signal.
- TV norm $\|\cdot\|_{\mathrm{TV}}$ is given by the ℓ_1-norm of the gradient of the signal.
- Tolerance ϵ is related to an estimate of the noise variance.

Spherical radio interferometric imaging

- Extend the standard compressed sensing imaging framework to wide fields by considering **interferometric images directly on the sphere**, rather than the equatorial plane.
- Augment the usual interferometric measurement operator with an initial projection $\mathbf{P}$ from the sphere to the plane, *i.e.*

$$\boldsymbol{y} = \Phi_\text{s}\, \boldsymbol{x}_\text{s} + \boldsymbol{n}, \quad \text{where} \quad \Phi_\text{s} = \Phi\, \mathbf{P}.$$

- Projection incorporates convolutional gridding on the sphere to afford use of FFTs
  (*cf.* gridding of continuous to discrete visibilities).
- Careful attention given to sampling densities to ensure accurate representation of band-limited signals:
  - Small FoV $\Rightarrow$ $L \simeq 2\pi B$
  - Wide FoV $\Rightarrow$ $L_\text{FoV} \simeq 2\pi \cos(\theta_\text{FoV}/2) B_\text{FOV}$
- Spherical interferometric images recovered by solving the BP or TV denoising problems, replacing measurement operator Φ with its spherical equivalent Φ_s.

Spherical radio interferometric imaging

- Extend the standard compressed sensing imaging framework to wide fields by considering **interferometric images directly on the sphere**, rather than the equatorial plane.
- Augment the usual interferometric measurement operator with an initial projection $\mathbf{P}$ from the sphere to the plane, *i.e.*

$$\boldsymbol{y} = \Phi_{\mathrm{s}}\, \boldsymbol{x}_{\mathrm{s}} + \boldsymbol{n}, \quad \text{where} \quad \Phi_{\mathrm{s}} = \Phi\, \mathbf{P}.$$

- Projection incorporates convolutional gridding on the sphere to afford use of FFTs
(*cf.* gridding of continuous to discrete visibilities).
- Careful attention given to sampling densities to ensure accurate representation of band-limited signals:
  - Small FoV $\Rightarrow$ $L \simeq 2\pi B$
  - Wide FoV $\Rightarrow$ $L_{\mathrm{FoV}} \simeq 2\pi \cos(\theta_{\mathrm{FoV}}/2) B_{\mathrm{FOV}}$
- Spherical interferometric images recovered by solving the BP or TV denoising problems, replacing measurement operator Φ with its spherical equivalent Φ_{s}.

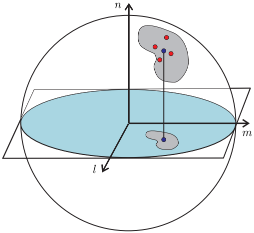


Figure: Projection operator.

Spherical radio interferometric imaging

- Extend the standard compressed sensing imaging framework to wide fields by considering **interferometric images directly on the sphere**, rather than the equatorial plane.
- Augment the usual interferometric measurement operator with an initial projection $\mathbf{P}$ from the sphere to the plane, *i.e.*

$$\boldsymbol{y} = \Phi_{\rm s}\, \boldsymbol{x}_{\rm s} + \boldsymbol{n}, \quad \text{where} \quad \Phi_{\rm s} = \Phi\, \mathbf{P}.$$

- Projection incorporates convolutional gridding on the sphere to afford use of FFTs
(*cf.* gridding of continuous to discrete visibilities).
- Careful attention given to sampling densities to ensure accurate representation of band-limited signals:
  - Small FoV $\Rightarrow$ $L \simeq 2\pi B$
  - Wide FoV $\Rightarrow$ $L_{\rm FoV} \simeq 2\pi \cos(\theta_{\rm FoV}/2) B_{\rm FOV}$
- Spherical interferometric images recovered by solving the BP or TV denoising problems, replacing measurement operator Φ with its spherical equivalent $\Phi_{\rm s}$.

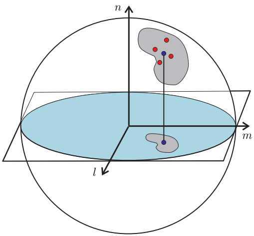


Figure: Projection operator.

Spherical radio interferometric imaging

- Extend the standard compressed sensing imaging framework to wide fields by considering **interferometric images directly on the sphere**, rather than the equatorial plane.
- Augment the usual interferometric measurement operator with an initial projection $\mathbf{P}$ from the sphere to the plane, *i.e.*

$$\boldsymbol{y} = \Phi_{\rm s}\, \boldsymbol{x}_{\rm s} + \boldsymbol{n}, \quad \text{where} \quad \Phi_{\rm s} = \Phi\, \mathbf{P}.$$

- Projection incorporates convolutional gridding on the sphere to afford use of FFTs
  (*cf.* gridding of continuous to discrete visibilities).
- Careful attention given to sampling densities to ensure accurate representation of band-limited signals:
  - Small FoV $\Rightarrow$ $L \simeq 2\pi B$
  - Wide FoV $\Rightarrow$ $L_{\rm FoV} \simeq 2\pi \cos(\theta_{\rm FoV}/2) B_{\rm FOV}$
- Spherical interferometric images recovered by solving the BP or TV denoising problems, replacing measurement operator Φ with its spherical equivalent $\Phi_{\rm s}$.

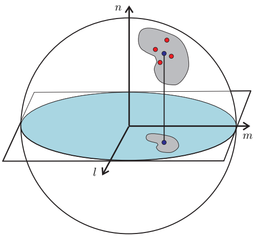


Figure: Projection operator.

Intrinsic advantages

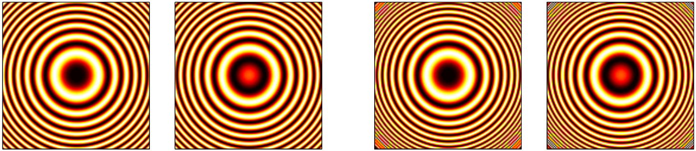
- Performance of compressed sensing reconstruction driven by sparsity and coherence.
- **Enhance both sparsity and coherence** in the wide-field spherical imaging framework.
- By recovering interferometric images on the sphere, distorting projections are eliminated and the number of samples required to represent a band-limited signal is reduced
  $\rightarrow$ **sparsity enhanced** $\rightarrow$ **fidelity of reconstructed image improved**.
- Spread spectrum (SS) phenomenon is enhanced on wide fields.
  - The w component induces the SS modulation, spreading spectrum of the signal.
  - Fourier measurements: coherence is max modulus of FT of sparsity basis vectors.
  - Spreading spectrum increases incoherence between sensing and sparsity bases.
  - **Wider FoV** $\rightarrow$ **high frequency content in** w **component modulation** $\rightarrow$ **more effective SS phenomenon** $\rightarrow$ **fidelity of reconstructed image improved.**

Intrinsic advantages

- Performance of compressed sensing reconstruction driven by sparsity and coherence.
- **Enhance both sparsity and coherence** in the wide-field spherical imaging framework.
- By recovering interferometric images on the sphere, distorting projections are eliminated and the number of samples required to represent a band-limited signal is reduced
  $\rightarrow$ **sparsity enhanced** $\rightarrow$ **fidelity of reconstructed image improved**.
- Spread spectrum (SS) phenomenon is enhanced on wide fields.
  - The w component induces the SS modulation, spreading spectrum of the signal.
  - Fourier measurements: coherence is max modulus of FT of sparsity basis vectors.
  - Spreading spectrum increases incoherence between sensing and sparsity bases.
  - **Wider FoV $\rightarrow$ high frequency content in w component modulation $\rightarrow$ more effective SS phenomenon $\rightarrow$ fidelity of reconstructed image improved.**

Intrinsic advantages

- Performance of compressed sensing reconstruction driven by sparsity and coherence.
- **Enhance both sparsity and coherence** in the wide-field spherical imaging framework.
- By recovering interferometric images on the sphere, distorting projections are eliminated and the number of samples required to represent a band-limited signal is reduced
  $\rightarrow$ **sparsity enhanced** $\rightarrow$ **fidelity of reconstructed image improved**.
- Spread spectrum (SS) phenomenon is enhanced on wide fields.
  - The w component induces the SS modulation, spreading spectrum of the signal.
  - Fourier measurements: coherence is max modulus of FT of sparsity basis vectors.
  - Spreading spectrum increases incoherence between sensing and sparsity bases.
  - **Wider FoV** $\rightarrow$ **high frequency content in** w **component modulation** $\rightarrow$ **more effective SS phenomenon** $\rightarrow$ **fidelity of reconstructed image improved.**

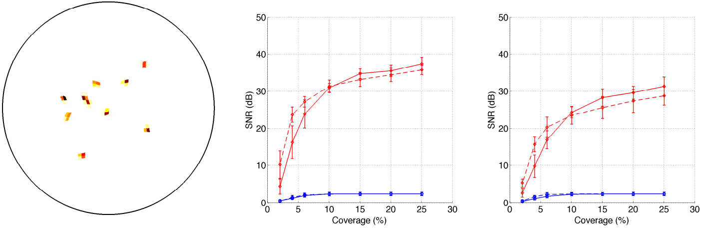


(a) Assuming $\|l\|^4 \, w \ll 1$ (b) No small-field assumption

Figure: Real part and imaginary part of SS modulation for FoV $\theta_{\mathrm{FoV}} = 90^\circ$.

Reconstruction of simulated Gaussian maps

- Quantify performance on simulations of Gaussian sources of various size for FoV $\theta_{\rm FoV} = 90^\circ$.

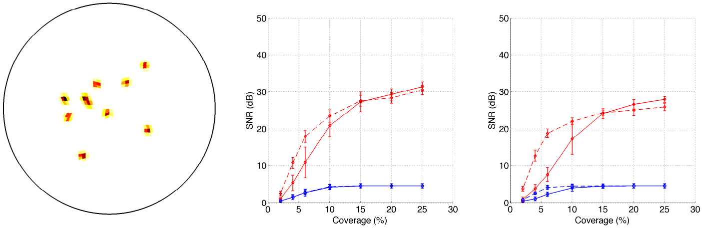


(a) Spherical image (b) $\rm SNR_s$ for BP (c) $\rm SNR_s$ for TV

Figure: Reconstruction performance for $\sigma_{\rm S} = 0.01$ (blue = plane; red = sphere; solid = no SS; dashed = SS).

Reconstruction of simulated Gaussian maps

- Quantify performance on simulations of Gaussian sources of various size for FoV $\theta_{\mathrm{FoV}} = 90^\circ$.

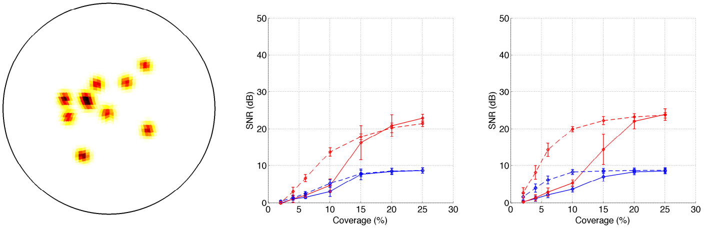


(a) Spherical image (b) $\mathrm{SNR_s}$ for BP (c) $\mathrm{SNR_s}$ for TV

Figure: Reconstruction performance for $\sigma_{\mathrm{S}} = 0.02$ (blue = plane; red = sphere; solid = no SS; dashed = SS).

Reconstruction of simulated Gaussian maps

- Quantify performance on simulations of Gaussian sources of various size for FoV $\theta_{\mathrm{FoV}} = 90^\circ$.

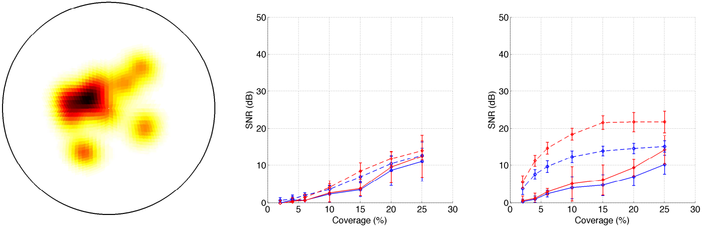


(a) Spherical image (b) $\mathrm{SNR_s}$ for BP (c) $\mathrm{SNR_s}$ for TV

Figure: Reconstruction performance for $\sigma_{\mathrm{S}} = 0.04$ (blue = plane; red = sphere; solid = no SS; dashed = SS).

Reconstruction of simulated Gaussian maps

- Quantify performance on simulations of Gaussian sources of various size for FoV $\theta_{\mathrm{FoV}} = 90^\circ$.

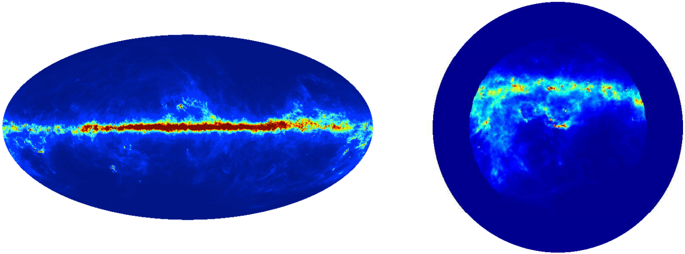


(a) Spherical image (b) $\mathrm{SNR_s}$ for BP (c) $\mathrm{SNR_s}$ for TV

Figure: Reconstruction performance for $\sigma_S = 0.10$ (blue = plane; red = sphere; solid = no SS; dashed = SS).

Reconstruction of Galactic dust map

- Consider more realistic, higher resolution simulation of 94GHz FDS map of predicted submillimeter and microwave emission of diffuse interstellar Galactic dust (Finkbeiner *et al.* 1999) (available form LAMBDA website: `http://lambda.gsfc.nasa.gov`).
- Reconstruct FoV $\theta_{\rm FoV} = 90^\circ$ from 25% of visibilities.

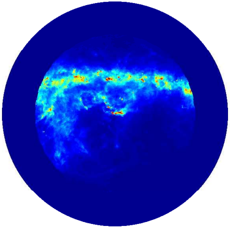


(a) Mollweide projection of full-sky

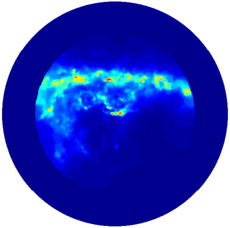
(b) Orthographic projection of FOV

Figure: FDS map of predicted emission of diffuse interstellar Galactic dust.

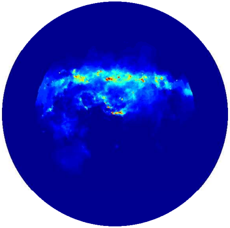
Reconstruction of Galactic dust map



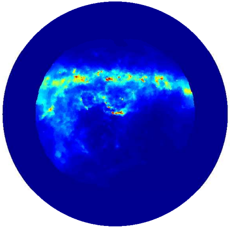
(a) Ground truth



(b) Planar reconstruction with SS (14dB)



(c) Spherical reconstruction without SS (7dB)



(d) Spherical reconstruction with SS (19dB)

Figure: Simulated TV reconstructions of diffuse FDS map.

Summary & future work

- **Spherical radio interferometric imaging**: solve inverse problem on the sphere.
- Enhances both sparsity and coherence:
  - **Sparsity:** eliminate distorting projections and reduce number of samples required to represent band-limited signal.
  - **Coherence:** spread spectrum phenomenon more effective on wide fields.

  $\rightarrow$ **improves fidelity of recovered interferometric images**.
- Current techniques idealised in order to remain as close as possible to the theoretical compressed sensing setting.
- Now that the effectiveness of these techniques has been demonstrated, it is of paramount importance to adapt them to **realistic interferometric configurations**.
- Consider continuous visibilities due to realistic interferometric configurations..
- Study the spread spectrum phenomenon in the presence of varying w.

LTS² ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Summary & future work

- **Spherical radio interferometric imaging**: solve inverse problem on the sphere.
- Enhances both sparsity and coherence:
  - **Sparsity:** eliminate distorting projections and reduce number of samples required to represent band-limited signal.
  - **Coherence:** spread spectrum phenomenon more effective on wide fields.

  $\rightarrow$ **improves fidelity of recovered interferometric images**.
- Current techniques idealised in order to remain as close as possible to the theoretical compressed sensing setting.
- Now that the effectiveness of these techniques has been demonstrated, it is of paramount importance to adapt them to **realistic interferometric configurations**.
- Consider continuous visibilities due to realistic interferometric configurations..
- Study the spread spectrum phenomenon in the presence of varying w.

LTS2 ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Summary & future work

- **Spherical radio interferometric imaging**: solve inverse problem on the sphere.
- Enhances both sparsity and coherence:
  - **Sparsity:** eliminate distorting projections and reduce number of samples required to represent band-limited signal.
  - **Coherence:** spread spectrum phenomenon more effective on wide fields.

  $\rightarrow$ **improves fidelity of recovered interferometric images**.
- Current techniques idealised in order to remain as close as possible to the theoretical compressed sensing setting.
- Now that the effectiveness of these techniques has been demonstrated, it is of paramount importance to adapt them to **realistic interferometric configurations**.
- Consider continuous visibilities due to realistic interferometric configurations..
- Study the spread spectrum phenomenon in the presence of varying w.

Summary & future work

- **Spherical radio interferometric imaging**: solve inverse problem on the sphere.
- Enhances both sparsity and coherence:
  - **Sparsity:** eliminate distorting projections and reduce number of samples required to represent band-limited signal.
  - **Coherence:** spread spectrum phenomenon more effective on wide fields.

  $\rightarrow$ **improves fidelity of recovered interferometric images**.
- Current techniques idealised in order to remain as close as possible to the theoretical compressed sensing setting.
- Now that the effectiveness of these techniques has been demonstrated, it is of paramount importance to adapt them to **realistic interferometric configurations**.
- Consider continuous visibilities due to realistic interferometric configurations..
- Study the spread spectrum phenomenon in the presence of varying w.