Fast directional spherical wavelets for CMB analysis

7 April 2005

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CSWT

Theory

- Follow formulation of Antoine and Vandergheynst (1999)
- Construct wavelet basis on the sphere
  - Extend motions (=rotations) and dilations to the sphere
  - Construct mother spherical wavelets

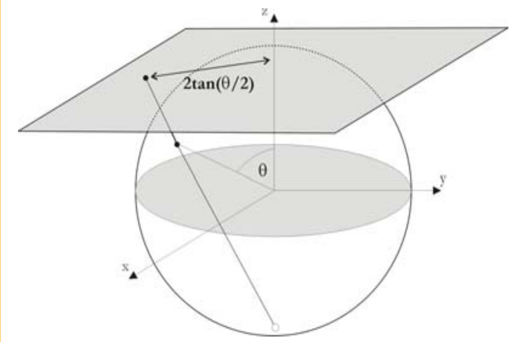
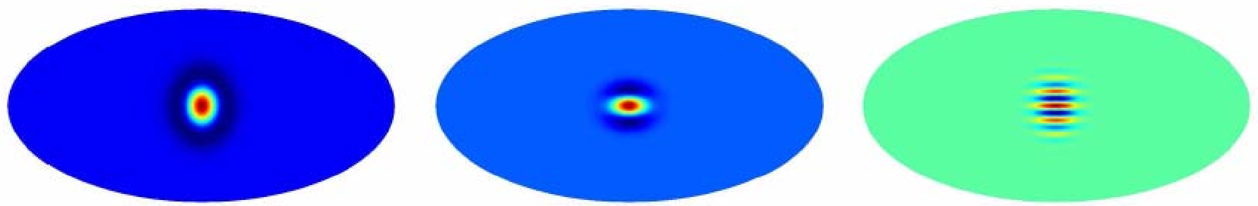


Figure 1. Stereographic projection of the sphere onto the plane.



(a) Mexican hat $\epsilon = 0.00$ (b) Mexican hat $\epsilon = 0.95$ (c) Real Morlet $\boldsymbol{k} = (10, 0)^T$

Figure 2. Spherical wavelets (dilation $a = 750'$; size on sky $\xi = 2105'$).

- Inherently directional (but local in nature) astro-ph/0409288

- Based on fast spherical convolution algorithm of Wandelt and Gorski (2001)
- Harmonic formulation with factored rotation
- Pose in form suitable for FFTs
- Savings

Algorithm	Complexity
Direct	$\mathcal{O}(L^4 N_\gamma)$
Semi-fast	$\mathcal{O}(L^3 \log_2(L) N_\gamma)$
Fast	$\mathcal{O}(L^3 N_\gamma)$

$\Rightarrow$ Saving: $\mathcal{O}(L) \sim \mathcal{O}(\sqrt{N})$

$\mathrm{N_{side}}$	Execution time (min:sec)		
	Direct	Semi-fast	Fast
8	00:01.19	00:01.12	00:00.01
16	00:18.60	00:17.38	00:00.04
32	05:01.48	04:43.06	00:00.21
256	-	-	01:54.15

Sun Fire 280R Server, Dual UltraSPARC III 900MHz Processors, 4GB Memory

Non-Gaussianity

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- Ability to probe different scales, positions and orientations
- CSWT linear: Gaussian sky $\rightarrow$ Gaussian coefficients
- Look for deviations in skewness and kurtosis of wavelet coefficients
  (Construct and apply extended coefficient exclusion masks)
- 1000 Monte Carlo simulations

Non-Gaussianity

Results

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- Compare coefficient statistics with Monte Carlo simulations
- Analyse most significant detections in more detail
- Significance tests

WMAP: 5.6σ

Tegmark: 6.4σ

WMAP: 98.3%

WMAP: 3.1σ

Tegmark: 3.2σ

WMAP: 95.3%

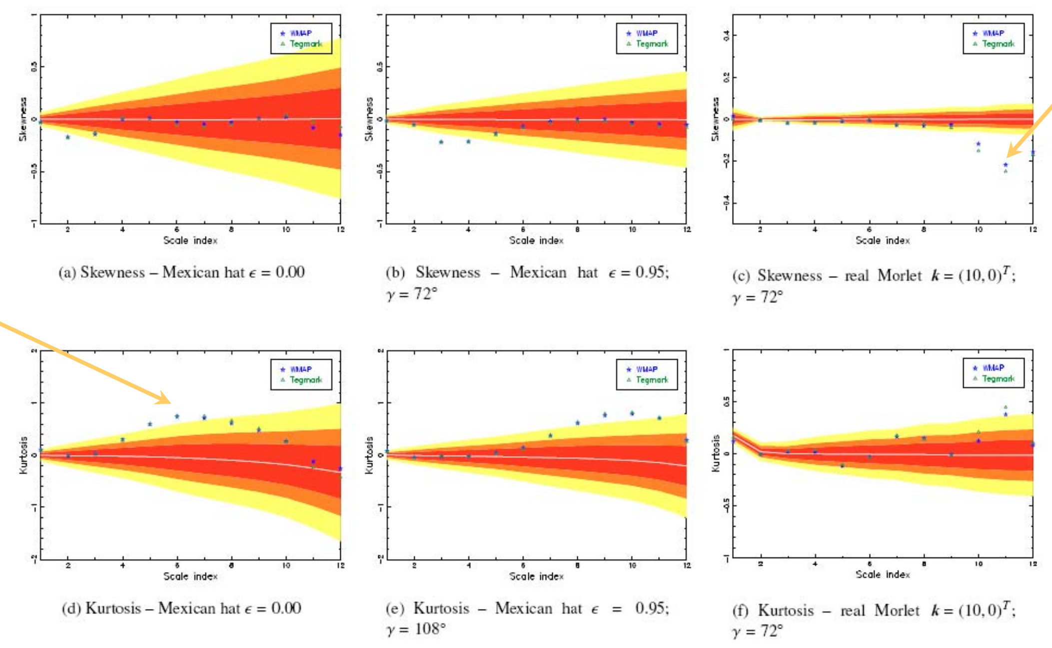

Figure 6. Spherical wavelet coefficient statistics for each wavelet. Confidence regions obtained from 1000 Monte Carlo simulations are shown for 68% (red), 95% (orange) and 99% (yellow) levels, as is the mean (solid white line). Only the orientations corresponding to the most significant deviations from Gaussianity are shown for the Mexican hat $\epsilon = 0.95$ and real Morlet wavelet cases.

Non-Gaussianity

Localised regions

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- Wavelets inherently provide spatial localisation
- Regions that introduce non-Gaussianity identified

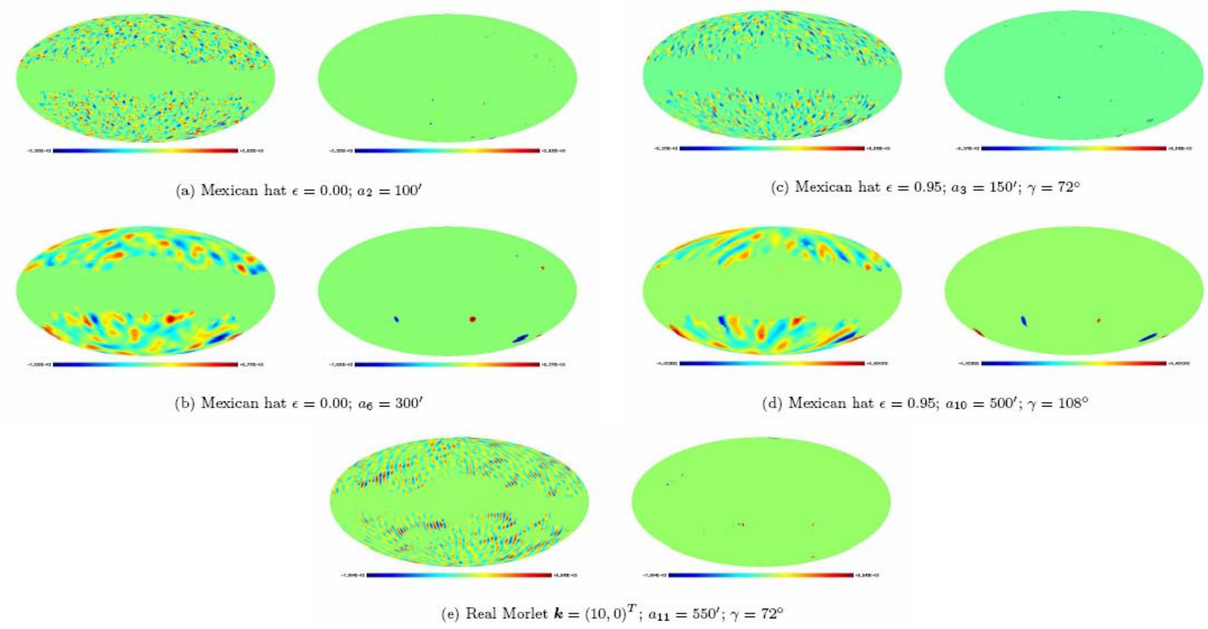

Figure 8. Spherical wavelet coefficient maps (left) and thresholded maps (right). To localise most likely deviations from Gaussianity on the sky, the coefficient maps exhibiting strong non-Gaussianity are thresholded so that only those coefficients above 3σ (in absolute value) are shown. Due to the apparent similarity of the WMAP team and Tegmark maps, only coefficients for the analysis of the WMAP map are shown above.

Other applications

- Any area where a wavelet analysis of full sky maps is required (i.e. where scale and spatial localisation beneficial)
- Examples:
  - Compact object detection

    (e.g. point sources, SZ effect, cosmic strings)
  - Late-time ISW effect

    (CMB-LSS cross-correlation in wavelet space)

Summary

- Fast directional CSWT
- Deviation from Gaussianity in WMAP 1-year data
  - Confirmed results obtained by Vielva et al. (2003)
  - More significant detections found using real Morlet wavelets at size on sky $\sim 26°/3°$
  - Localised regions
- Other applications
  - Compact object detection
  - ISW effect
  - others…