Detection of the ISW effect and corresponding dark energy constraints
(astro-ph/0602398)

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Outline

1. Integrated Sachs-Wolfe (ISW) effect
   - Physical origin
   - Detecting the effect

2. The continuous spherical wavelet transform (CSWT)
   - Dilations and mother wavelets on the sphere
   - Transform

3. Cross-correlation in wavelet space
   - Wavelet covariance estimator
   - Comparison of wavelets

4. Analysis procedure

5. Results
   - Detections
   - Dark energy constraints

6. Summary
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ISW effect
Physical origin

- Photons blue (red) shifted when fall into (out of) potential wells
- Evolution of potential during photon propagation → net change in photon energy
- Large scale phenomenon (cosmic variance limited → require full-sky maps)
- Only present in non-flat universes or flat universes with dark energy

Temperature perturbation

\[
\frac{\delta T}{T} = 2 \int \frac{\dot{\Phi}}{c^2} \frac{d\ell}{c}
\]

where \(d\ell\) is the element of proper distance. In Einstein de-Sitter universe (no \(\Lambda\)), \(\Phi_k \sim \delta_k/a\) and linear growth law for \(\Omega = 1\) is \(\delta_k \sim a\). Thus \(\dot{\Phi} \neq 0\) only when \(\Omega\) diverges significantly from unity.
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Detecting the ISW effect
Cross-correlating the CMB with LSS

- Cannot directly separate the ISW signal from CMB anisotropies
- Detected by cross-correlating CMB anisotropies with tracers of large scale structure (Crittenden & Turok 1996)
- Detections used to place constraints on dark energy
- Previous works
  - Real space angular correlation function (e.g. Boughn & Crittenden 2002)
  - Harmonic space cross-angular power spectrum (e.g. Afshordi et al. 2004)
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Summary
Spherical wavelet transform
Anisotropic dilation on the sphere

- Spherical wavelet transform (Antoine and Vandergheynst 1998; Wiaux et al. 2005)
- Stereographic projection $\Pi$
- Anisotropic dilation on the sphere

$$D(a, b) = \Pi^{-1} d(a, b) \Pi$$

$$[D(a, b)s](\omega) = [\lambda(a, b, \theta, \phi)]^{1/2} s(\omega_{1/a,1/b})$$

where

$$\omega_{a,b} = (\theta_{a,b}, \phi_{a,b})$$,

$$\tan(\theta_{a,b}/2) = \tan(\theta/2) \sqrt{a^2 \cos^2 \phi + b^2 \sin^2 \phi}$$

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Spherical wavelet transform
Mother wavelets on the sphere

- Stereographic projection of admissible Euclidean mother wavelets

\[ \psi(\omega) = [\Pi^{-1}\psi_{\mathbb{R}^2}](\omega) \]

Figure: Spherical wavelets at scale \( a = b = 0.2 \).
**Spherical wavelet transform**

- **Motion on the sphere (≡ rotation)**
  \[
  [R(\rho)s](\omega) = s(\rho^{-1}\omega), \quad \rho \in SO(3)
  \]

- **Multi-resolution basis on the sphere**
  \[
  \{\psi_{a,b,\rho} \equiv R(\rho)D(a,b)\psi; \quad \rho \in SO(3); \quad a, b \in \mathbb{R}_+^*\}
  \]

- **Spherical wavelet transform**
  \[
  W_\psi(a, b, \rho) \equiv \int_{S^2} d\Omega(\omega) \psi_{a,b,\rho}^*(\omega) s(\omega)
  \]

- **Fast algorithm** (McEwen et al. 2005; Wandelt & Gorski 2001)
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Wavelet covariance estimator

- Suitability of wavelets for detecting cross-correlations

- Wavelet covariance

\[
\hat{X}_{\psi}^{NT}(a, b, \gamma) = \frac{1}{N_{\alpha\beta}} \sum_{\alpha, \beta} \nu_{\alpha\beta} W_{\psi}^{N}(a, b, \alpha, \beta, \gamma) W_{\psi}^{T}(a, b, \alpha, \beta, \gamma)
\]

- Average over orientations

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\hat{X}_{\psi}^{NT}(a, b) = \frac{1}{N_{\gamma}} \sum_{\gamma} \hat{X}_{\psi}^{NT}(a, b, \gamma)
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- Theoretical wavelet covariance

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Comparison of wavelets

- Compare predicted signal-to-noise ratio

\[
\text{SNR}_\psi(a, b) = \frac{\langle \hat{X}^{NT}_\psi(a, b) \rangle}{\Delta \hat{X}^{NT}_\psi(a, b)}
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where

\[
\left[ \Delta \hat{X}^{NT}_\psi(a, b) \right]^2 = \sum_{\ell=0}^{\infty} \frac{1}{2\ell + 1} p_\ell^4 (b^N_\ell)^2 (b^T_\ell)^2 \left[ \sum_{m=-\ell}^{\ell} |(\psi_{a, b})_{\ell m}|^2 \right]^2 \left[ (C^{NT}_\ell)^2 + C^{TT}_\ell C^{NN}_\ell \right]
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- Similar technique used to compare real, harmonic and wavelet space techniques for detection of cross-correlations
  → wavelets optimal on certain scales (Vielva et al. 2006)
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SNR plots

(a) All wavelets
(b) SMHW
(c) SBW

Figure: Expected SNR of the wavelet covariance estimator of CMB and radio source maps

Don’t consider SMW further
(actually considered; as expected not effective)
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- Data
- Analysis (scales; masks)
- Simulations
- Constraints on dark energy parameters
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- **Data**

  ![Map of WMAP1 and NVSS](image)

- **Analysis (scales; masks)**
  - Simulations
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Scales and detections

- **Scales**

<table>
<thead>
<tr>
<th>Scale</th>
<th>1</th>
<th>2</th>
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<th>7</th>
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<td>Dilation a</td>
<td>100'</td>
<td>150'</td>
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<td>250'</td>
<td>300'</td>
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<td>500'</td>
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<tr>
<td>Size on sky 1</td>
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<tr>
<td>Size on sky 2</td>
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- **Wavelet covariance plots**
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- **Wavelet covariance plots**

  - SMHW
  - SMHW
  - SBW

Jason McEwen
Detection of the ISW effect
Most significant detections

- Wavelet covariance statistics appear Gaussian
  \( \rightarrow N_\sigma \) direct indication of significance of detections
  - symmetric SMHW: 3.6\( \sigma \); elliptical SMHW: 3.9\( \sigma \); SBW: 3.9\( \sigma \)

- \( N_\sigma \) plots (2 and 3\( \sigma \) contours)
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Systematics and foregrounds

- **Systematics:** individual WMAP receiver maps
  → systematics not likely source of detection

- **Foregrounds:** foreground dominated difference maps
  → foregrounds not likely source of detection

![Individual receiver maps](image1)

- SMHW
- SBW

![Difference maps](image2)

- SMHW
- SBW

Jason McEwen
Detection of the ISW effect
Localised regions
Detection

- Wavelets inherently provide spatial localisation (in addition to scale localisation)
- Threshold wavelet coefficient product maps to localise most likely sources
Wavelets inherently provide spatial localisation (in addition to scale localisation)

Threshold wavelet coefficient product maps to localise most likely sources

Symmetric SMHW  Elliptical SMHW  SBW
Remove localised regions $\rightarrow$ ISW detection remains
(Agrees with findings of Boughn and Crittenden 2004)

- Symmetric SMHW
- Elliptical SMHW
- SBW

Examined localised regions in closer detail
Compute theoretical wavelet covariance for range of models \((w, \Omega_\Lambda)\)
(assume concordance model for other parameters; bias \(b = 1.6\))

- Compare theoretical predictions with observations

\[
\chi^2(w, \Omega_\Lambda) = \Delta^T C^{-1} \Delta
\]

where

\[
\Delta = [\hat{X}_\psi^{NT}(a, b, \gamma) - X_\psi^{NT}(a, b, \gamma|w, \Omega_\Lambda)]
\]

- Compute likelihood

\[
\mathcal{L}(w, \Omega_\Lambda) \propto \exp[-\chi^2(w, \Omega_\Lambda)/2]
\]
**Dark energy constraints**

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Parameter estimates from mean of marginalised distributions

- $\Omega_\Lambda = 0.63^{+0.18}_{-0.17}$, $w = -0.77^{+0.35}_{-0.36}$ using SMHW
- $\Omega_\Lambda = 0.52^{+0.20}_{-0.20}$, $w = -0.73^{+0.42}_{-0.46}$ using SBW
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- $\Omega_\Lambda = 0.52^{+0.20}_{-0.20} \; ; \; w = -0.73^{+0.42}_{-0.46}$ using SBW
Also considered case \( w = -1 \)

- \( \Omega_\Lambda = 0.70^{+0.15}_{-0.15} \) using SMHW
- \( \Omega_\Lambda = 0.57^{+0.18}_{-0.18} \) using SBW

Reject \( \Omega_\Lambda = 0 \) at > 99% significance

- \( \Omega_\Lambda > 0.1 \) at 99.9% using SMHW
- \( \Omega_\Lambda > 0.1 \) at 99.7% using SBW
Also considered case $w = -1$

- $\Omega_\Lambda = 0.70^{+0.15}_{-0.15}$ using SMHW
- $\Omega_\Lambda = 0.57^{+0.18}_{-0.18}$ using SBW

Reject $\Omega_\Lambda = 0$ at > 99% significance

- $\Omega_\Lambda > 0.1$ at 99.9% using SMHW
- $\Omega_\Lambda > 0.1$ at 99.7% using SBW
Outline

1. Integrated Sachs-Wolfe (ISW) effect
   - Physical origin
   - Detecting the effect

2. The continuous spherical wavelet transform (CSWT)
   - Dilations and mother wavelets on the sphere
   - Transform

3. Cross-correlation in wavelet space
   - Wavelet covariance estimator
   - Comparison of wavelets

4. Analysis procedure

5. Results
   - Detections
   - Dark energy constraints

6. Summary
Summary

- Used spherical wavelets to detect ISW effect
- Detection of ISW effect made at almost $4\sigma$ → effectiveness of wavelets
- Foregrounds and systematics not likely source of detection
- Independent evidence of dark energy
- Consistent constraints on dark energy
  - Good consistency check with direct estimates from other approaches
  - Wavelets of similar performance for constraining dark energy as other ISW techniques