Analysing the polarisation of the CMB with spin scale-discretised wavelets

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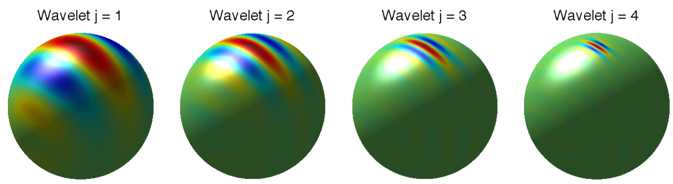
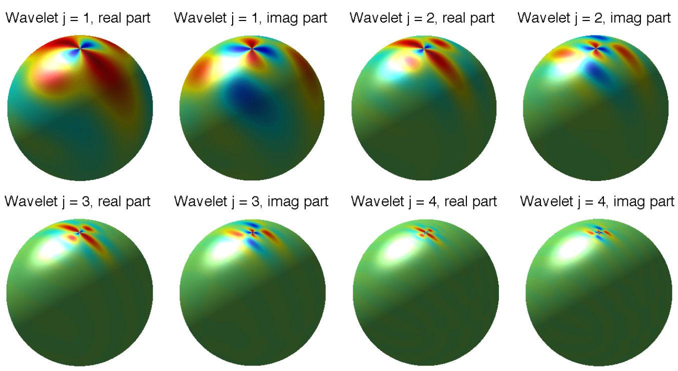
***Abstract*—We discuss a new scale-discretised directional wavelet transform to analyse spin signals defined on the sphere, in particular the polarisation of the cosmic microwave background (CMB).**

I. Spin scale-discretised wavelets on the sphere

We design a directional scale-discretised wavelet transform to analyse the directional features of signals of arbitrary spin on the sphere. Following [1], [2], [3], [4], the spin-discretised wavelets ${}_s\Psi^j$ on the sphere are defined in harmonic space as ${}_s\Psi^j_{\ell m} \equiv \kappa^j(\ell)\xi_{\ell m}$, where $\kappa^j(\ell)$ characterises the angular localisation of the wavelets (for the jth scale), while $\xi_{\ell m}$ controls their directionality. Fig. 1 shows examples of wavelets obtained with this construction.



(a) Spin-0 scale-discretised (directional) wavelets



(b) Spin-2 scale-discretised (directional) wavelets

Fig. 1. Spin 0 and 2 wavelets constructed with angular $L = 512$, azimuthal band-limit $N = 15$, and tilling parameters $B = 2$ and $J_{\min} = 2$.

The wavelet transform of a spin signal ${}_sf$ on the sphere is given by the directional convolution with the wavelets. Provided the wavelets satisfy an admissibility property, the original signal can be synthesised exactly from its wavelet coefficients. More details about this transform can be found in [4].

II. Application to the CMB

The polarisation of the CMB is currently an intense avenue of research, since it may reveal signatures of primordial gravitational waves and a glimpse on the initial conditions of the universe. However, the physical quantities necessary for these investigations – namely the E- and B-modes of the polarisation, or gradient and curl modes – are not directly available. They are obtained by reducing and transforming frame-dependent observables: the Q and U "local" polarisation measured on the sky. This (Q,U) to (E,B) transformation is unambiguous when dealing with data covering the entire sky. However, CMB observations cover fractions of the sky only, and (E,B) reconstruction is imperfect near the boundaries of the observation mask, causing leakage and potential biases in the recovered E- and B-mode maps. This issue is traditionally addressed by smoothing and extending the mask to remove boundary regions where the leakage is important. We develop a method exploiting the novel directional spin wavelet transform to obtain a more accurate EB reconstruction over a great area of the sky.

First, we compute the wavelet coefficients of the observable $Q+iU$ using our *spin* wavelet transform. Second, we mitigate the impact of partial-sky data in spin wavelet space. Third, we apply to the real and imaginary parts of these wavelet coefficients a *scalar* inverse wavelet transform. It can be shown that this yields estimates of the E and B signals provided the wavelets of this inverse transform are spin-lowered versions of those used in the first transform [4]. This new method to estimate the E- and B-mode contributions from Q and U observations can significantly reduce the E-B leakage by exploiting improved masking in wavelet space.

Our novel spin wavelet transform will be of general use in analysing CMB polarisation data. A more detailed description of spin scale-discretised wavelets, fast algorithms, and a rigorous evaluation of their performance will be given in a series of forthcoming articles.

References

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