Wavelet-Based Segmentation Method for Spherical Images

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***Abstract*—In this work, we review a new wavelet-based method [1] proposed to segment images on the sphere, accounting for the underlying geometry of spherical data. The method is compatible with any arbitrary type of wavelet frame defined on the sphere, such as axisymmetric wavelets, directional wavelets, curvelets, and hybrid wavelet constructions. Numerical experiments on projected spherical retina images demonstrate the superior performance of the proposed method.**

I. Introduction

Segmentation is the process of identifying object outlines within images. There are a number of efficient algorithms for segmentation in Euclidean space that depend on the variational approach and partial differential equation modelling, e.g. [2], [3]. Wavelets have been used successfully in various problems in image processing, including segmentation, inpainting, noise removal, and many others. Wavelets on the sphere have been developed to solve such problems for data defined on the sphere, which arise in numerous fields such as cosmology and geophysics, e.g. [4], [5], [6].

We review the wavelet-based spherical image segmentation method proposed in [1], which is a direct extension of the tight-frame based segmentation method [3] used to automatically identify tube-like structures such as blood vessels in medical imaging. It is compatible with any arbitrary type of wavelet frame defined on the sphere. Such an approach allows the desirable properties of wavelets to be naturally inherited in the segmentation process. Moreover, the algorithm is efficient with convergence usually within a few iterations.

The segmentation method devised in [1] provides, for the first time, a segmentation framework for spherical images. The framework used an iterative strategy with the flexibility to tailor the iterative procedure according to data types and features.

II. Algorithm

Let $f \in \mathrm{L}^2(\mathbb{S}^2)$ be the given image defined on the sphere $\mathbb{S}^2$. Without loss of generality, we assume f in [0, 1]. Let $\mathcal{A}$ and $\mathcal{A}^{-1}$ be the forward and backward spherical wavelet transforms, respectively.

The idea behind the method is to detect the candidates of possible pixels on (near) the boundary first, then gradually purify these boundary-like pixels via an iterative procedure until all pixels on the sphere are classified as inside or outside of a boundary. When a binary result is obtained the algorithm stops. In the following, we discuss each of the iterative steps of the method in more detail.

Preprocessing. Suppress the noise in f by using one iteration step of the tight-frame algorithm in [3], then represent it by $\bar{f}$.

Initialisation. Let $\Lambda^{(0)}$ be the initial set of potential boundary pixels, which is identified by using the gradient of $\bar{f}$, i.e. pixels with gradient larger than a given threshold ϵ are in $\Lambda^{(0)}$.

The i-th iteration of our algorithm can be described by: 1) find a range $[a_i,\, b_i]$ from image $f^{(i)}$ and threshold it into three parts – those below, inside and above the range (represented by $f^{(i+\frac{1}{2})}$), and obtain $\Lambda^{(i+1)}$ which contains fewer potential boundary pixels; 2) compute a new image using the following formula

$$f^{(i+1)} \equiv (\mathcal{I} - \mathcal{P}^{(i+1)}) f^{(i+\frac{1}{2})} + \mathcal{P}^{(i+1)} \mathcal{A}^{-1} \mathcal{T}_\lambda (\mathcal{A} f^{(i+\frac{1}{2})}),$$

where $\mathcal{T}_\lambda$ represents the soft-thresholding with threshold λ; $\mathcal{I}$ is the identity operator and $\mathcal{P}^{(i+1)}$ is the operator generated from $\Lambda^{(i+1)}$.

Stopping criterion. As soon as all the pixels of $f^{(i+\frac{1}{2})}$ are either of value 0 or 1, or equivalently when $\Lambda^{(i)} = \emptyset$, the iteration is terminated, then all the pixels with value 1 constitute the objects of interest otherwise they are considered as background.

III. Experimental results

Experimental results of a spherical retina image are given in Fig. 1, to demonstrate the superiority of the method and show its capability of segmenting spherical images, including those with prominent directional features. The test image is generated by projecting a 2D retina image in the DRIVE data-set[1] on the sphere.

The K-means method is applied to data on the sphere according to the pixels intensities for comparison purpose, using the MATLAB built-in function `kmeans`. The result by the method [1] equipping hybrid wavelets constructed by combining the directional wavelets and curvelets is obtained with $\epsilon = 0.04$. Code to compute these wavelet transforms is public and available in the existing `S2LET`[2] package.

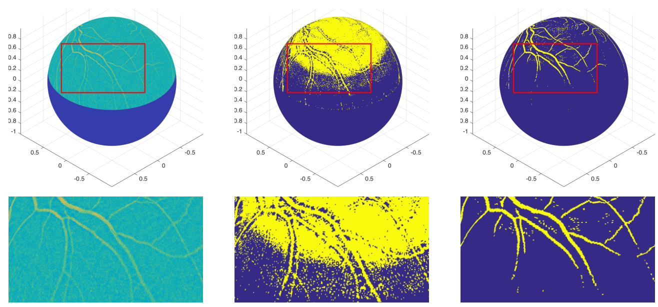

Fig. 1. Results of spherical retina image. First row from left to right gives the spherical retina image, the segmentation result of K-means method and that of the method in [1] (takes 8 iterations); with the zoomed-in details of the red rectangle areas on them shown in the second row, respectively.

References

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