

# COMPRESSED SENSING FOR RADIO INTERFEROMETRIC IMAGING: REVIEW AND FUTURE DIRECTION

Jason D. McEwen<sup>1</sup> & Yves Wiaux<sup>1,2,3</sup>

<sup>1</sup>Institute of Electrical Engineering, Ecole Polytechnique Fédérale de Lausanne (EPFL), Switzerland

<sup>2</sup>Institute of Bioengineering, Ecole Polytechnique Fédérale de Lausanne (EPFL), Switzerland

<sup>3</sup>Department of Radiology & Medical Informatics, University of Geneva (UniGE), Switzerland

## ABSTRACT

Radio interferometry is a powerful technique for astronomical imaging. The theory of compressed sensing (CS) has been applied recently to the ill-posed inverse problem of recovering images from the measurements taken by radio interferometric telescopes. We review novel CS radio interferometric imaging techniques, both at the level of acquisition and reconstruction, and discuss their superior performance relative to traditional approaches. In order to remain as close to the theory of CS as possible, these techniques necessarily consider idealised interferometric configurations. To realise the enhancement in quality provided by these novel techniques on real radio interferometric observations, their extension to realistic interferometric configurations is now of considerable importance. We also chart the future direction of research required to achieve this goal.

**Index Terms**— Compressed sensing, sparsity, radio interferometry, interferometric imaging

## 1. INTRODUCTION

Aperture synthesis interferometry is a powerful technique in radio astronomy, providing astronomical images at angular resolution and sensitivity that are otherwise inaccessible (see *e.g.* [1]). Arrays of telescopes track a source as it traverses the sky to synthesise the aperture of a telescope of much larger size than the individual telescopes comprising the array. Radio interferometric arrays directly probe the Fourier plane associated with the image tangent to the celestial sphere at the source position. As the source traverses the sky, interferometric measurements, termed visibilities, are generated on an elliptical track of positions in the Fourier plane. Visibility coverage in the Fourier plane is therefore incomplete. Consequently, recovering interferometric images from measured

visibilities requires solving an ill-posed deconvolution problem.

The recovery of interferometric images has traditionally been tackled by the so-called CLEAN algorithm [2], which is a local matching pursuit approach. Recently, however, the theory of compressed sensing (CS) [3, 4] has been applied to interferometric imaging with considerable success, both at level of acquisition and reconstruction [5, 6, 7]. Furthermore, CS techniques have been developed to successfully extract astronomical signals of interest from interferometric observations corrupted by background contributions [8]. First steps in these directions necessarily involved the study of idealised interferometric configurations to remain as close to the theory of CS as possible. With a new generation of interferometric telescopes under design and construction, radio interferometry will continue to play a fundamental role in astronomy. The extension of CS techniques to more realistic interferometric configurations is therefore now of paramount importance, in order to realise the enhancement in quality that these techniques provide on real observations made by radio interferometric telescopes.

In this article we review the current state-of-the-art of CS imaging techniques for radio interferometry. We then chart the future direction that this research must take to be applicable to real radio interferometric telescopes. In Sec. 2 we review radio interferometry concisely and pose the interferometric imaging inverse problem. We discuss the CS techniques applied to this problem in Sec. 3, before charting the future direction of research in Sec. 4.

## 2. INTERFEROMETRIC INVERSE PROBLEM

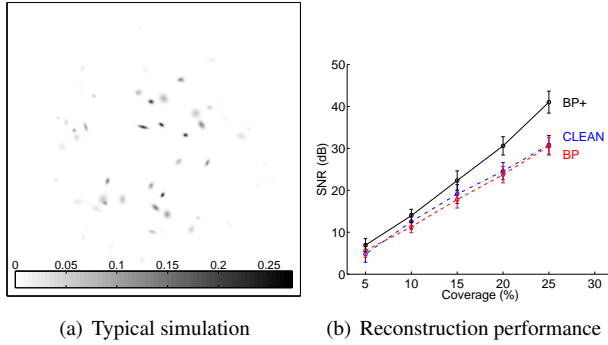
The field-of-view (FoV) observed by an interferometer is limited by the primary beam of the telescope  $A$ . We denote the sky intensity to be imaged by  $x$  and consider a coordinate system centred on the pointing direction of the telescope  $\hat{s}_0$ , where a point  $\hat{s}$  on the celestial sphere is defined by its coordinates  $(l, m, n)$ , noting  $n \equiv n(\mathbf{l}) = (1 - \|\mathbf{l}\|_2^2)^{1/2}$ , for  $\mathbf{l} = (l, m)$  and  $\|\mathbf{l}\|_2^2 = l^2 + m^2$ . This configuration is illustrated in Fig. 1. The complex visibility measured by the cor-

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URL: [www.jasonmcewen.org](http://www.jasonmcewen.org)

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**Fig. 2.** BP, BP+ and CLEAN reconstruction performance on elongated Gaussian profile signals (credit: [5]).

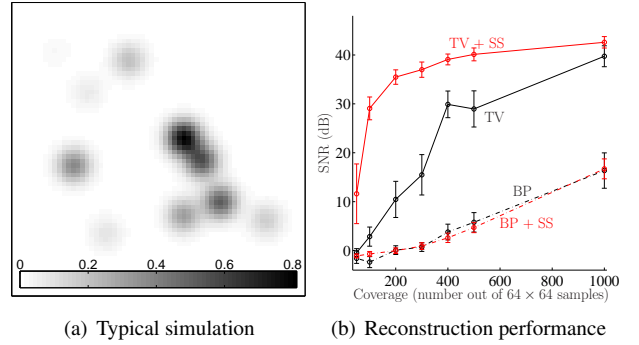
### 3.2. Spread spectrum phenomenon

If small FoV assumptions are not made, then the  $w$ -modulation modulation  $\mathbf{C}$  no longer reduces to the identity. Beyond sparsity, CS demonstrates that the incoherence between the measurement and sparsity bases is a key criterion driving the performance of CS based reconstructions. In the interferometric setting the measurement basis is essentially the Fourier basis, hence coherence is given by the maximum modulus of the Fourier coefficient of the sparsity basis vectors. Consequently, an operation that acts to reduce the maximum Fourier coefficient, reduces the coherence and thus improves the quality of CS reconstructions. The  $w$ -modulation corresponds to a norm-preserving convolution in the Fourier plane, spreading the spectrum of the sparsity basis vectors, and achieves exactly this. The Dirac basis considered previously is already maximally incoherent, hence this spread spectrum phenomenon is not expected to improve performance for this case. However, for alternative bases we expect an enhancement.

Many natural signals are also sparse or compressible in the magnitude of their gradient, in which case the Total Variation (TV) minimisation problem applies [3]. The TV problem involves replacing the  $\ell_1$ -norm sparsity prior in the BP problem with a prior on the TV norm of the signal itself:

$$\min_{\mathbf{x}} \|\mathbf{x}\|_{\text{TV}} \text{ such that } \|\mathbf{y} - \Phi\mathbf{x}\|_2 \leq \epsilon,$$

where the TV norm is defined by the  $\ell_1$ -norm of the gradient of the signal  $\|\mathbf{x}\|_{\text{TV}} = \|\nabla\mathbf{x}\|_1$ . The TV problem may also be solved by the application of non-linear, iterative convex optimisation algorithms, illustrating another example of the versatility of the framework. The signal is directly recovered from the solution to the optimisation problem. Recovering interferometric images by solving the TV problem has been shown to achieve very good reconstruction quality and, moreover, is enhanced considerably by the spread spectrum phenomenon [6]. A typical simulation used to study this effect and reconstruction quality for BP (with Dirac sparsity)



**Fig. 3.** BP and TV reconstruction performance on Gaussian profile signals, in the absence and presence of the spread spectrum (SS) phenomenon (credit: [6]).

and TV approaches, in the absence and presence of the spread spectrum phenomenon, are shown in Fig. 3.

### 3.3. Wide field-of-view

Incorporating wide FoV considerations in interferometric imaging is of increasing importance for the next generation of radio interferometric telescopes. CS techniques for interferometric imaging have been extended to a wide FoV recently [7], recovering images in the spherical coordinate space in which they naturally live. The effectiveness of the spread spectrum phenomenon is enhanced when going to a wide FoV since the  $w$ -modulation evolves to include higher frequency content, while sparsity is promoted by recovering images directly on the sphere. Both of these properties act to improve the quality of reconstructed interferometric images.

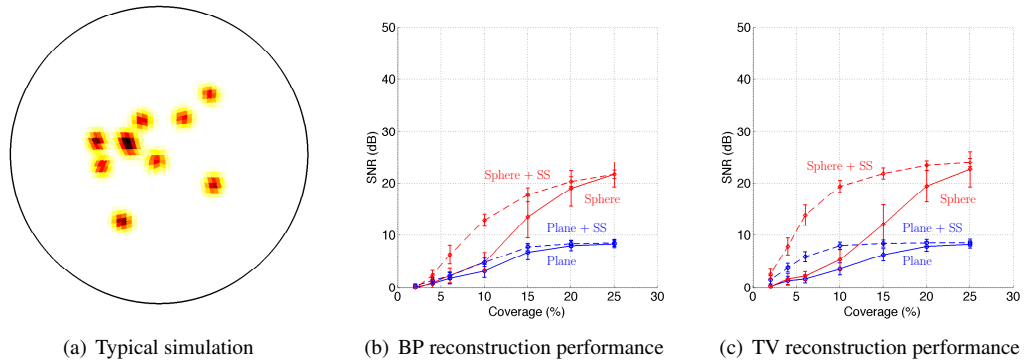
The wide FoV ill-posed inverse problem is defined by

$$\mathbf{y} = \Phi_s \mathbf{x}_s + \mathbf{n}, \quad (3)$$

with

$$\Phi_s = \mathbf{MFCAP},$$

where the vector  $\mathbf{x}_s$  is the interferometric image defined on the celestial sphere (rather than a tangent plane). The measurement operator on the sphere  $\Phi_s$  simply consists of augmenting the operator on the plane  $\Phi$  by prepending a projection  $\mathbf{P}$  from the sphere to the plane, which incorporates a convolutional regridding. Careful consideration is given to samplings on the sphere and plane to ensure that the planar grid is sampled sufficiently to accurately represent the projection of a band-limited signal defined on the sphere. The BP and TV problems are then solved to recover interferometric images on the sphere. This approach has been shown to perform very well, with the quality of recovered spherical interferometric images superior to planar reconstructions [7]. A typical simulation used to study this effect and reconstruction quality for BP (with Dirac sparsity) and TV approaches both on the planar and spherical settings, in the absence and presence of the spread spectrum phenomenon, are shown in Fig. 4.



**Fig. 4.** BP and TV reconstruction performance on Gaussian profile signals on the plane and sphere, in the absence and presence of the spread spectrum (SS) phenomenon (credit: (author?) [7]).

#### 4. FUTURE DIRECTION

The CS techniques developed for interferometric imaging [5, 6, 7] remain somewhat idealised. Random visibility coverage in  $(u, v)$  is assumed, with the spatial frequencies probed by the interferometer also assumed to fall on discrete grid points. Furthermore, to study the spread spectrum phenomenon a constant  $w$  is assumed. These restrictions have been necessary to remain as close to the theory of CS as possible during the development and evaluation of interferometric imaging techniques. However, now that the effectiveness of these techniques has been demonstrated, it is important to adapt them to realistic interferometric configurations.

In practice a varying  $w$  will not alter the essence of the spread spectrum phenomenon due to its realisation through the convolution operator; however, this effect must be studied. The  $w$ -projection algorithm [9] may be applied to reduce the computational cost in this context.

Realistic visibility coverages in all components  $(u, v, w)$  must also be considered by simulating observations made by real interferometer configurations. Furthermore, to integrate CS imaging techniques into the data processing pipelines of radio interferometric telescopes, it is necessary to consider the continuous visibilities measured by an interferometer before they are gridded (so that imaging can be integrated into the iterative self-calibration of telescopes).

In general, CS addresses imaging by optimising both reconstruction and acquisition, while current applications in interferometry have essentially focused on reconstruction only. The possibility of optimising the configuration of interferometers to enhance the spread spectrum phenomenon [6] for CS reconstruction is an exciting avenue of research at the level of acquisition, contrary to the traditional approach that attempts to minimise this effect. In addition, direction dependent beam effects may also provide an alternative source of the spread spectrum phenomenon.

In conclusion, with a new generation of radio interferometric telescopes under design and construction, such as the

Square Kilometer Array (SKA), the development of novel interferometric techniques is of paramount importance. It is essential that the CS interferometric imaging techniques developed recently are adapted to realistic settings to ensure that the fidelity of reconstructed images keeps pace with the capabilities of new instruments.

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