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Bianchi $\mathrm{VII_h}$ models and the cold spot texture

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ABSTRACT
We have returned to our previous Bianchi $\mathrm{VII_h}$ analysis in light of the Cruz et al. (2007b) suggestion that the cold spot observed near the southern Galactic pole may be a remnant temperature perturbation of a cosmic texture. In Bridges et al. (2007) we found two favoured left handed Bianchi $\mathrm{VII_h}$ templates with restricted prior probabilities so that the template was centred close to the cold spot. Using WMAP data 'corrected' for the texture fit we have now reexamined both models to assess any changes to these conclusions. We find that both models are left almost entirely unconstrained by the data and consequently exhibit significantly reduced Bayesian evidences. Both models are now *disfavoured* by the data. This result reinforces our previous suggestion that the cold spot was likely to be driving any Bianchi $\mathrm{VII_h}$ detection.

Key words: cosmic microwave background – methods: numerical – methods: data analysis.

1 INTRODUCTION

The anomalous cold spot discovered (Vielva et al. 2004; Cruz et al. 2005, 2006, 2007a) in Wilkinson Microwave Anisotropy Probe (WMAP) observations of the cosmic microwave background (CMB) (Bennett et al. 2003; Hinshaw et al. 2007) has been shown by Cruz et al. (2007b) to be consistent with a temperature perturbation induced by a cosmic texture. A cosmic texture is a particular type of cosmic defect predicted by certain theories of high energy physics. They form at symmetry breaking phase transitions in the early Universe and are extremely energetic events producing both hot and cold spots in the CMB. The texture hypothesis is the most convincing explanation of the cold spot anomaly made to date, however evidence for the texture requires further observational support. One of the most promising aspects of the texture hypothesis is that tests may be performed using future observations of CMB polarisation, which may either substantiate or refute evidence for the presence of a texture. The discovery of a texture would have such profound implications for our understanding of the early Universe that the cold spot-texture hypothesis certainly warrants further investigation.

Bianchi $\mathrm{VII_h}$ models in which the universe exhibits a global rotation and shear have also been considered in an attempt to explain many of the anomalies in the WMAP data. A positive detection of a Bianchi $\mathrm{VII_h}$ template embedded in the WMAP data was first made by Jaffe et al. (2005). After 'correcting' the WMAP data for this template a number of previously reported anomalies in the data disappear (Jaffe et al. 2005; Land & Magueijo 2006; Cayon et al. 2006; McEwen et al. 2006). However, the corresponding best-fit Bianchi $\mathrm{VII_h}$ template was shown to be incompatible with concordance cosmology (Jaffe et al. 2006). Bridges et al. (2007) performed a more rigorous Bayesian MCMC analysis to determine the evidence for Bianchi $\mathrm{VII_h}$ models, concluding that there is weak evidence *against* a Bianchi $\mathrm{VII_h}$ component when the axis of the Bianchi coordinate system is allowed to vary over the entire sky. However, weak evidence *for* a Bianchi template remains when the axis is restricted to lie in the direction of the cold spot. These results suggest that it may have been the cold spot that was driving the positive template detection made by Jaffe et al. (2005).

The focus of this work is to determine whether there is evidence for any Bianchi $\mathrm{VII_h}$ component embedded in the WMAP data once the data is 'corrected' for the cold spot texture template determined by Cruz et al. (2007b). If all positive evidence vanishes, and one accepts the texture explanation of the cold spot, then Bianchi $\mathrm{VII_h}$ models may be rejected definitively. The remainder of this letter is structured as follows. Firstly, the procedure followed by Cruz et al. (2007b) to fit a texture template to the cold spot in the combined, foreground-cleaned Q-V-W map (hereafter WMAP co-added map) is discussed, before the template is used to 'correct' the WMAP internal linear combination (ILC) map (since the subsequent MCMC analysis requires a full-sky map and the co-added map necessitates a Galactic cut). Using different processed versions of the WMAP data for template fitting and analysis is acceptable and was shown by McEwen et al. (2006) not to alter analysis results for the Bianchi case. Secondly, the Bayesian evidence for the texture 'corrected' WMAP ILC map is computed and compared

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to the evidence computed previously (Bridges et al. 2007) for the original data. Concluding remarks are then made.

2 COLD SPOT TEXTURE FITTING

The cold spot has been shown by Cruz et al. (2007b) to be consistent with a temperature perturbation induced in the CMB by cosmic texture at redshift $z \approx 6$. Unwinding events associated with texture induce cold and hot spots in the CMB. An analytic approximation for the temperature profile produced by a texture is given by (Turok & Spergel 1990)

$$\frac{\delta T}{T} = \frac{\pm\epsilon}{\sqrt{1+4(\theta/\theta_C)^2}}, \quad (1)$$

where θ denotes angular separation, θ_C is the scale parameter of the texture and ϵ is the amplitude parameter related to the symmetry breaking scale ϕ_0. This temperature profile is an approximation and is not valid for large co-moving scales. Consequently, for practical applications the profile is truncated beyond its half-maximum by matching its value and derivative to a Gaussian, as discussed by Magueijo (1995). θ_C is then equal to the standard deviation of the matching Gaussian. The resulting texture temperature profile was fitted to the cold spot by Cruz et al. (2007b) and found to be favoured to the null hypothesis of no texture. In this section we review the texture fitting procedure performed by Cruz et al. (2007b) and present the best-fit texture profile determined.

The co-added map of the three-year WMAP data release (Hinshaw et al. 2007) is used to fit the texture profile to the cold spot. Cruz et al. (2007b) degrade the co-added map in the HEALPix[1] pixelization scheme (Górski et al. 2005) to a resolution parameter of $N_{\rm side} = 64$. This resolution is sufficient to retain all information associated with the cold spot, which occurs on a half-width scale of $\sim 5°$, and reduces the number of pixels used in the template fitting, thereby reducing the computational cost of the fitting procedure. The template fitting was performed in a circular area of 20° radius centred at Galactic coordinates ($b = -57°, l = 209°$). Although the total angular size of the cold spot is $\sim 10°$, it is necessary to consider a 20° radius patch in order to take into account the entire neighbourhood of the spot since the original Spherical Mexican Hat Wavelet analyses that highlighted the spot (Vielva et al. 2004) convolves all pixels in this region. These pixels could therefore contribute in an important way to the detected structure and must be included when fitting the texture template.

A hybrid Bayesian-frequentist approach is considered for the template fitting performed by Cruz et al. (2007b). The Bayesian evidence ratio is used to perform hypothesis testing, which is then calibrated using Monte Carlo simulations. The data are found to favour the alternative hypothesis H_a that a texture is present. Parameters ϵ and θ_C of the temperature template profile $\mathbf{T}$, defined by (1), are then determined by maximising the posterior probability for the alternative hypothesis, where the posterior is given by Bayes' Theorem:

$$Pr(\mathbf{\Theta}|\mathbf{D}, H_a) \propto Pr(\mathbf{D}|\mathbf{\Theta}, H_a) Pr(\mathbf{\Theta}|H_a), \quad (2)$$

with likelihood $Pr(\mathbf{D}|\mathbf{\Theta}, H_a)$ and prior $Pr(\mathbf{\Theta}|H_a)$, where $\mathbf{D}$ represents the WMAP co-added data and $\mathbf{\Theta} = (\epsilon, \theta_C)$. The likelihood function is assumed to be Gaussian:

$$L \propto e^{-\frac{\chi^2}{2}}, \quad (3)$$

where

$$\chi^2 = (\mathbf{D}-\mathbf{T})^{\rm T}\mathbf{N}^{-1}(\mathbf{D}-\mathbf{T}), \quad (4)$$

with the generalised noise covariance matrix $\mathbf{N}$ including both CMB and noise contributions. The calculation of the noise contribution to $\mathbf{N}$ is straightforward since the noise in the WMAP co-added map is uncorrelated and well defined through the number of observations per pixel. In order to obtain the CMB contribution to the matrix, the covariance function for the WMAP co-added map is calculated taking into account both pixel and beam effects. As a complementary test the CMB covariance matrix is calculated through 70,000 Gaussian simulations. Cruz et al. (2007b) compared the χ^2 values computed from the data to those obtained using simulations and found errors were negligible. As a conservative prior on ϵ, Cruz et al. (2007b) choose $0 \leqslant \epsilon \leqslant 10^{-4}$, the COBE-normalised amplitude (Pen et al. 1994; Durrer et al. 1999). A scale-invariant distribution of texture spots on the sky is predicted

$$\frac{dN_{spot}}{d\theta_C} = \frac{8\pi\nu}{3}\frac{\kappa^3}{\theta_C^3}, \quad (5)$$

where ν is a dimensionless constant and κ a fraction of unity. In order to obtain the prior for the scale parameter θ_C, (5) is normalised to unity between $\theta_{\rm min}$ and $\theta_{\rm max}$. Photon diffusion would smear out textures smaller than a degree or so, hence the lower θ_C bound is set to $\theta_{\rm min} = 1°$, according to the resolution considered. At large scales textures are rare, hence the upper θ_C bound is set to $\theta_{\rm max} = 15°$. In this setting Cruz et al. (2007b) obtain texture parameter estimates of $\epsilon = 7.7 \times 10^{-5}$ and $\theta_C = 5.1°$. The original co-added data, the best-fit texture template and their difference are shown on a small patch on the sky in Fig 1. of Cruz et al. (2007b).

3 EVIDENCE FOR BIANCHI VII$_h$ MODELS

Bianchi VII$_{\rm h}$ models induce characteristic 'spiral' temperature fluctuations on the CMB sky. They are described by four independent quantities: a matter energy density $\Omega_{\rm m}$, a dark energy density Ω_Λ, the current vorticity ω and h, which physically relates the characteristic wavelength over which the principle axes of shear and vorticity change orientation, and determines the 'tightness' of the spiral pattern. The pattern position is defined by Euler angles α, β and γ [2]. Additionally one must specify the direction of rotation or handedness of the spiral.

Jaffe et al. (2005) found evidence of a significant correlation between a Bianchi VII$_{\rm h}$ model and the WMAP 1-year data. In Bridges et al. (2007) we performed a more rigorous Bayesian analysis that confirmed the Jaffe et al. (2005) fit in WMAP 3-year data. In this framework our entire inference is contained in the multidimensional posterior distribution from which we can extract marginalised parameter constraints *and* the comparative Bayesian evidence to select the most appropriate model parameterisation. We aimed to use the evidence to establish whether it was necessary to include a Bianchi VII$_{\rm h}$ component in addition to a standard ΛCDM cosmology. We concluded that the only significant Bayesian evidence favouring such an inclusion existed where the central position of the Bianchi VII$_{\rm h}$ spiral was fixed close to the cold spot. In

[1] http://healpix.jpl.nasa.gov/

[2] We adopt the active *zyz* Euler convention corresponding to the rotation of a physical body in a *fixed* coordinate system about the z, y and z axes by γ, β and α respectively.

Table 1. Summary of Bianchi VII$_h$ component priors used in this analysis.

Full Bianchi
$\Omega^{B}_{tot} = [0.01, 0.99]$
$\Omega^{B}_{m} = [0.01, 0.99]$
$h = [0.01, 1]$
$\omega = [0, 20] \times 10^{-10}$
$\gamma = [0, 2\pi]$rads
Chirality = L/R

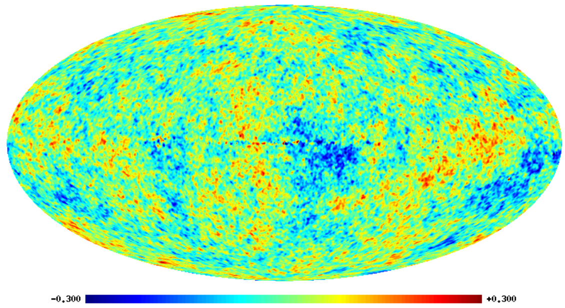
Table 2. Cosmological and Bianchi parameterisations for each of the parameter subsets studied.

Model	Cosmology	Bianchi
A	A_s	-
D	A_s	Ω^{B}_{m}, Ω^{B}_{tot}, h, ω, γ, L/R
G	A_s	Ω^{B}_{m}, h, ω, γ, L/R

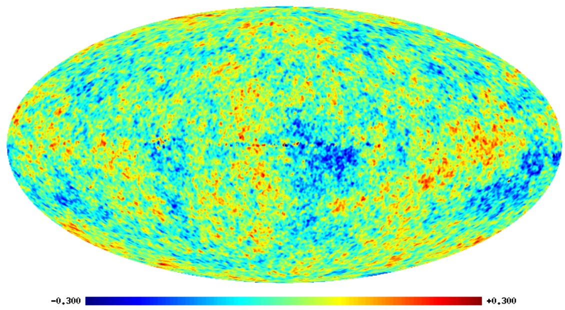
this letter we aim to establish how the significance of this conclusion has changed in light of the texture fit of Cruz et al. (2007b). We use the WMAP 3-year ILC map (Fig. 1 (a)) subtracting the cold spot template described by Cruz et al. (2007b) (Fig. 1 (c)) to yield the 'corrected' map shown in Fig. 1 (b).

The models we will consider here are those of A, D & G from Bridges et al. (2007), for convenience we summarise the parameter combinations and priors in Tables 1 & 2. Model A is a standard ΛCDM cosmology, with one free parameter A_s, the amplitude of scalar fluctuations, this being the only parameter constrained on these large scales. Both models D and G had the Bianchi template axis fixed close to the cold spot via heavily constrained priors on α and β. However D and G differ in the inclusion of a dark energy density component which significantly opens up the parameter space along a degeneracy in the $\Omega_\Lambda - \Omega_m$ plane (see Fig. 6 in Bridges et al. 2007). It was first suggested by Jaffe et al. (2006) that a dark energy term might give the Bianchi VII$_h$ models more freedom to accomodate a close to flat cosmology preferred by WMAP and others. Although the Bianchi degeneracy found is intriguingly close to the geometric degeneracy seen in Spergel et al. (2007) it does not cross the WMAP confidence contours above the 2σ level. From this we concluded that the set of cosmological parameters preferred by WMAP are incompatible with those of any Bianchi VII$_h$ component. We proceeded in our analysis by decoupling the Bianchi VII$_h$ matter and energy densities from their ΛCDM counterparts so that the Bianchi VII$_h$ component was essentially treated as a template with energy densities Ω^{B}_{m} and Ω^{B}_{Λ}. Left-handed models D and G were the only ones found to show marginally significant evidence favouring their inclusion and so we have chosen these to carry over for this analysis. Morphologically both D and G are very similar (owing to the degeneracy described above); the best fitting template is shown in Fig. 1 (d).

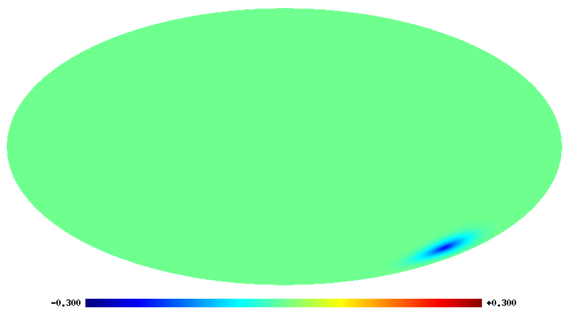
In Bridges et al. (2007) both models D and G resulted in highly constrained marginalised posterior distributions (red lines of Fig. 2 & 3) on each of the Bianchi VII$_h$ parameters. In this analysis (black lines of Fig. 2 & 3) however all parameters except perhaps a slight preference for $h \sim 0.2$, are entirely unconstrained, and crucially, a non-zero likelihood is observed in the vorticity at $\omega = 0$. In the Bianchi VII$_h$ formalism ω is highly correlated with Bianchi VII$_h$ signal amplitude so such a result illustrates that with the texture



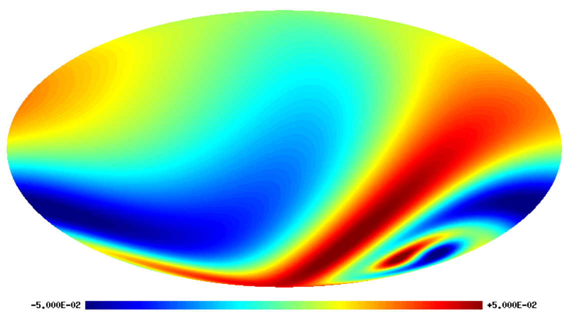
(a) WMAP ILC map



(b) Cold spot corrected WMAP ILC map



(c) Cold spot template



(d) Previous best-fit Bianchi template

Figure 1. Full-sky CMB and correction maps displayed in the Mollweide projection. The original WMAP ILC map is illustrated in panel (a), while the cold spot corrected map is shown in panel (b). The cold spot template is displayed on a full-sky map in panel (c). In panel (d) the best-fit Bianchi VII$_h$ template determined by Bridges et al. (2007) is shown. Notice that the cold spot texture template aligns closely with the central cold swirl of the best-fit Bianchi template. All maps are given in units of mK.

Table 3. Bayesian evidence differences (logarithmic) between models A and left-handed D & G for the original WMAP 3-year ILC map and that corrected for the texture fit of Cruz et al. (2007b).

Data \ Model	D (L)
3-year WMAP	+1.2 ± 0.2
3-year WMAP (texture corrected)	-0.1 ± 0.2
Data \ Model	G (L)
3-year WMAP	+1.2 ± 0.2
3-year WMAP (texture corrected)	-0.8 ± 0.2

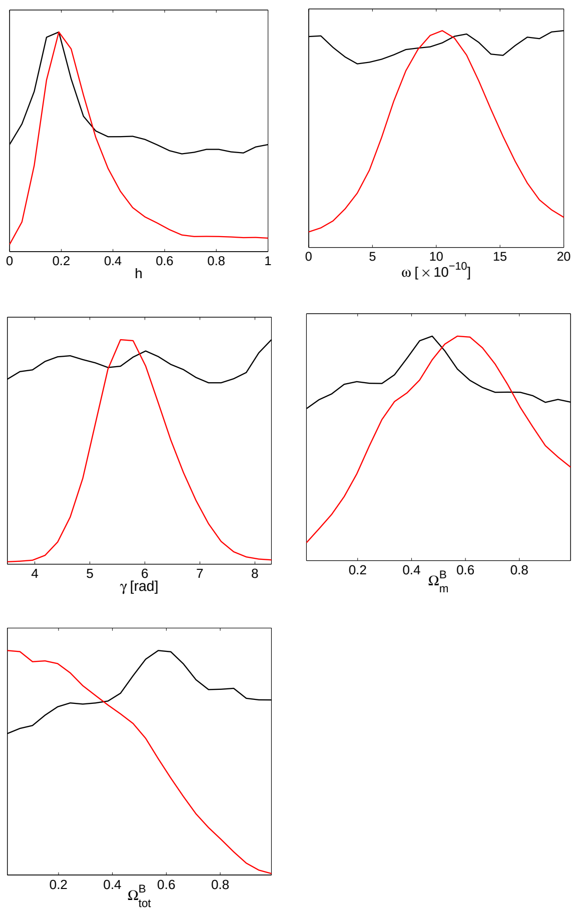


Figure 2. Marginalised Bianchi VII_h parameters for left-handed model D using the original WMAP ILC map (red) and that corrected for the texture fit of Cruz et al. (2007b) (black).

corrected map the data no longer prefers any additional components of this kind over a standard cosmology. Although redundant, given the unconstrained posteriors, the same conclusions can be confirmed via the Bayesian evidence estimates (see Table 3). Both models D and G record significantly reduced evidences, model D is now about as favoured as a ΛCDM cosmology alone (model A) while model G is now, almost significantly, *disfavoured* when compared with A. In Bridges et al. (2007) all right-handed models were found to have disfavouring evidences. It is worth noting that this result remains unchanged with the texture corrected data. Both left and right handed models are now equally disfavoured.

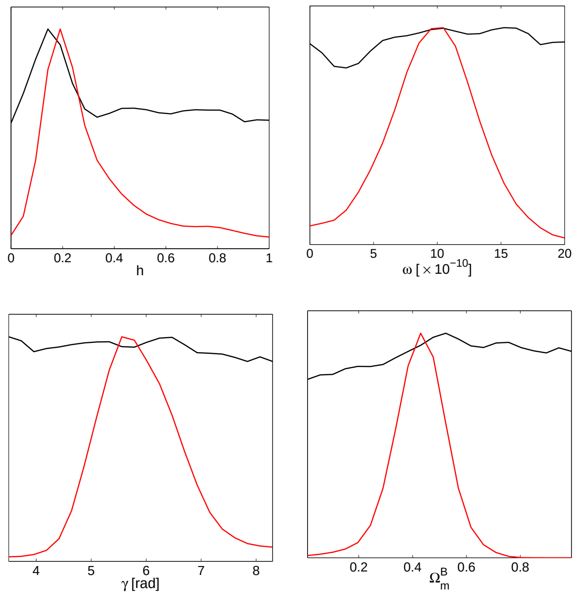


Figure 3. Marginalised Bianchi VII_h parameters for left-handed model G using the original WMAP ILC map (red) and that corrected for the texture fit of Cruz et al. (2007b) (black).

4 CONCLUSIONS

In Bridges et al. (2007) we concluded that the preferred Bianchi VII_h template was likely to be heavily influenced by the cold spot in the southern galactic hemisphere. Accepting the recent texture fit (Cruz et al. 2007b) essentially removes this feature from the WMAP data. Although there were other discernible features that appeared reduced following 'correction' of the data by the best-fit Bianchi VII_h template, these were mostly concentrated close to the galactic plane where little confidence can be placed in the map quality. The Bayesian evidence in favour of the inclusion of a Bianchi VII_h component, was only marginally significant for just two of the models we had previously considered. Using data 'corrected' for the texture, both of these models are now left almost entirely unconstrained by the data and consequently register disfavouring log evidence values. This result raises further doubts over the relevance of a Bianchi VII_h explanation for any of the anomalous features seen in the current WMAP data.

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